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VICTOR HUGO RESENDE LIMA

A NEW PROBLEM FOR SELECTIVE MAINTENANCE CONSIDERING BI-OBJECTIVES, REPAIRPERSON ASSIGNMENT AND K-OUT-OF-N SYSTEMS

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Dissertation presented to the Graduate Program in Industrial Engineering of Federal University of Pernambuco, as partial requirement to obtain the master’s degree in Industrial Engineering.

Concentration Field: Operational Research.

Advisor: Prof. Dr. Cristiano Alexandre Virgínio Cavalcante.

Co-Advisor: Prof. Dr. Raphael Harry Frederico Ribeiro Kramer.

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ABSTRACT

This dissertation deals with the maintenance optimization problem in a multi-component system, which should undergo maintenance actions between two consecutive missions, preparing itself for the next mission. Due to time, budget and resource limitations, top-level actions cannot be performed on all components and therefore, a subset of components and actions should be selected for the objective optimization. Most of the existing models to tackle this kind of problem do not involves complex systems or, when they do it, they consider only one objective to be optimized. To study the establishment of problems that consider complex systems, multi-objective approaches and repairperson assignments, this work proposes a new non-linear binary model that models the bi-Objective Selective Maintenance and Repairperson Assignment Problem on *k*-out-of-*n* systems (bi-OSMRAP:*k*-out-of-*n*). Its modeling is discussed, and three algorithms are proposed for the problem solving: a full enumeration algorithm, a metaheuristic and a matheuristic, these last two based on the *Adaptive Variable Neighborhood Search*. Two instances were tested, one artificial instance and the other from the literature, and a sensitive analysis was conducted to understand the problem behavior. Both approximated algorithms were solid, supported by good values for the metrics used.

Keywords: selective maintenance; *k*-out-of-*n* systems; metaheuristic; matheuristic; combinatorial optimization.

RESUMO

Esta dissertação trata do problema de otimização de manutenção em um sistema multi-componente, o qual deve passar por ações de manutenção entre duas missões consecutivas, preparando-o para a próxima missão. Devido aos limites no tempo, orçamento e recursos, ações de alta qualidade não podem ser feitas em todos componentes e portanto, um sub-conjunto de componentes e ações devem ser selecionados para a otimização do objetivo. A maioria dos modelos existentes para resolver este tipo de problema não envolve sistemas complexos, ou quando tratam com esse tipo de sistema, eles só consideram um objetivo a ser otimizado. Para estudar o estabelecimento de problemas que consideram sistemas complexos, abordagens multi-objetivo e designações de mantenedores, este trabalho propõe um novo modelo não-linear binário que modela o Problema bi-Objetivo de Manutenção Seletiva e Designação de Mantenedores em sistemas *k-out-of-n.* Toda a modelagem é discutida e três algoritmos são propostos para a resolução do problema: um algoritmo de enumeração completa, uma metaheurística e uma *matheuristic*, sendo estes dois últimos baseados na *Adaptive Variable Neighborhood Search*. Duas instâncias foram testadas, uma artificial e outra oriunda da literatura, e uma análise de sensibilidade foi conduzida para elucidar o comportamento do problema. Ambos algoritmos aproximados se mostraram robustos, suportados por bons valores para as métricas usadas.

Palavras-chave: manutenção seletiva; Sistemas *k*-out-of-*n;* metaheurística; matheuristic; otimização combinatória.

FIGURE LIST

TABLE LIST

SYMBOL LIST

s	Number of subsystems
s_i	Number of components in subsystem i
n	Total number of components in the system
E_{ij}	Component j of subsystem i
k_i	Minimum required number of working components in subsystem i
B_{ij}	Effective age of E_{ij} at the break beginning
A_{ij}	Effective age of E_{ij} at the break ending
X_{ij}	Operational status of E_{ij} at the break beginning
Y_{ij}	Operational status of E_{ij} at the break ending
L_{ij}	List of possible actions for E_{ij}
l	Index on L_{ij}
α_{ijl}	Age reduction factor of the corrective action l performed on E_{ij}
t_{ijl}^c	Spent time to execute the corrective action l on E_{ij}
δ_{ijl}	Age reduction factor of the preventive action l performed on E_{ij}
t_{ijl}^p	Spent time to execute the preventive action l on E_{ij}
m	Number of repairpersons
r	Index of repairperson
c_r^f	Fixed cost of hiring the repairperson r
c_r^v	Variable cost of using the repairperson r
Ψ	Break time
U	Next mission length
y_{ijlr}	Decision variable of executing action l on E_{ij} by repairperson r
z_r	Decision variable of hiring repairperson r
η_{ij}	Scale parameter of the Weibull distribution for E_{ij}
β_{ij}	Shape parameter of the Weibull distribution for E_{ij}
$R_{ij}^c(U\|_{A_{ij}})$	Conditional reliability of E_{ij}, given U and effective age A_{ij}
$R_{ij}(t)$	Unconditional reliability of E_{ij} for a time t
R_i	Reliability for whole subsystem i
R	Reliability for the whole system
C	Solution cost
T_r	Spent time by repairperson r when he/she is working

$\Delta_{i,met}$	Number of iterations for the initial Pareto Frontier generation in the metaheuristic
$\tau_{i,met}$	Greedy factor for the initial Pareto Frontier generation in the metaheuristic
ρ_{met}	Probability to finish a single solution in the metaheuristic
$\Delta_{o,met}$	Number of iterations for the metaheuristic operation
$\tau_{o,met}$	Greedy factor in metaheuristic operation
μ_{met}	Maximum number of action/repairperson removals in one iteration of the metaheuristic
$\lambda_{init,met}$	Poisson distribution mean for the number of removals at the first iteration of the metaheuristic
$\lambda_{end,met}$	Poisson distribution mean for the number of removals at the last iteration of the metaheuristic
$\kappa_{init,met}$	Greedy factor for solution selecting at the first iteration of the metaheuristic
$\kappa_{end,met}$	Greedy factor for solution selecting at the last iteration of the metaheuristic
c_{ij}^{v}	Cost of repairperson *i* to execute the action on component *j*
WC_j	Existence of an action different from “Do-nothing” on component *j*
t_{ij}	Spent time by repairperson *i* executing the action on component *j*
x_{ij}	Decision variable for repairperson *i* to execute the action on component *j*
$\Delta_{i,math}$	Number of iterations for the initial Pareto Frontier generation in the matheuristic
$\tau_{i,math}$	Greedy factor for the initial Pareto Frontier generation in the matheuristic
ρ_{math}	Probability to finish a single solution in the matheuristic
$\Delta_{o,math}$	Number of iterations for the matheuristic operation
$\tau_{o,math}$	Greedy factor in matheuristic operation
μ_{math}	Maximum number of action/repairperson removals in one iteration of the matheuristic
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SUMMARY

1 INTRODUCTION

Maintenance is no longer seen as a resource expenditure after asset failures. Nowadays, it is recognized as an essential business function that provides a better connection with other functions within the company (DE JONGE and SCARF, 2020). As an example, Pinjala et al. (2006) presented some links between businesses and maintenance strategies. They analyzed different businesses characteristics, organizational structures of the maintenance, action features, technical complexity, teamwork, among others.

In reality, the maintenance function of an organization can be seen as its maintenance concept. This concept embraces the framework for the maintenance strategy selected by the organization. In other words, it is the "embodiment" of how the organization thinks about the role of the maintenance as an operation function (WAEYENBERGH and PINTELON, 2002). The establishment of the maintenance concept is a significant manner to optimize decisions in maintenance. For instance, Waeyenbergh and Pintelon (2004) implemented a maintenance concept on a cigar and cigarette factory, specifically in the bottleneck line of the plant, and the company's output increased almost by 100%.

Additionally, maintenance is a significant part of companies' total costs, representing from 15% to 70% of the total production costs (BEVILACQUA and BRAGLIA, 2000). In non-fossil-fuel energy generation plants, the maintenance represents the major portion of the total production cost (ZIO and COMPARE, 2013). Furthermore, in a opportunistic policy, Kang and Guedes Soares (2020) achieved good cost savings (41,9%) for offshore turbines. To summarize, the maintenance has a critical role in competitiveness improvement in organizations due to its portion on companies' total costs.

Age and block-based replacement and inspection policies (ZHAO et al., 2017; BARLOW and HUNTER, 1960; KAIO and OSAKI, 1986; BARLOW et al., 1963) are the most basic quantitative models for scheduling of programmed maintenance actions. However, these methods are not devoted to situations when the system is composed of several units and operates in a scheme of successive missions. Usually, systems have resource constraints, and therefore, it is not feasible to perform all top-level actions in their components. So, a subset of units and maintenance actions to be performed during the break between two missions should be selected to optimize an objective. This Combinatorial Optimization problem is the so-called Selective Maintenance Problem (SMP). The SMP describes some specific real problems faced by managers. For example, note that this problem is designed for systems with redundant components, i. e. although they have different characteristics, these components are functionality equals. In addition, SMPs are denoted for systems in which failures during the mission are unlikely.

SMP has been extensively studied (XU et al., 2015; CAO et al., 2018), since its first study in 1998 (RICE et al., 1998). However, there are some gaps in SMP literature, as Cao et al. (2018) indicated. For example, complex systems were not appropriately covered (Cao et al. (2018) indicate only eight studies). Concerning *k*-out-of-*n* systems, to the best of our knowledge, only one work dealt with this kind of system (DIALLO et al., 2018), despite its relevance to real-world applications, such as production and service systems.

On the other hand, a few gaps were fulfilled in recent years, e. g. selective maintenance and repairperson assignment problem. Khatab et al. (2018) proposed this joint problem considering imperfect actions, and then other works explored different problem features (KHATAB et al., 2019; CHAABANE et al., 2020). The impact of this joint decision is very relevant, and it can increase the maximum system reliability, despite the cost increase (KHATAB et al., 2018).

Usually, maintenance problems faced by companies involve more than one objective. For example, costs and reliability are two essential objectives considered by managers in this field (QUDDOOS et al., 2015; DIALLO et al., 2019). Because they are conflicting objectives, multi-objectives approaches are necessary. According to our literature review, some works deal with bi-objective modeling on SMP, but the works cited before (DIALLO et al., 2018; KHATAB et al., 2018) did not. Therefore, there is a lack of joint studies in multi-objective, repairperson assignment and complex systems.

Accordingly, this new problem tackles the maintenance optimization of systems composed of *k*-out-of-*n* subsystems, subjected to a maintenance crew where the manager wants to optimize the reliability and cost for the next system mission. This approach is designed for systems without a continuous operation where several missions need to be accomplished. Besides that, real situations where the reliability and costs are critical can be solved with this new problem.

1. 1 JUSTIFICATION

The SMP is an NP-hard problem (RICE, 1999), and there is still no efficient (polynomial) algorithm capable of solving large instances in a reasonable time. Given this feature, investigations in approximate algorithms are relevant, especially in metaheuristics. Cao et al. (2018) pointed out that a few metaheuristics were proposed for SMPs (Genetic

Algorithms, Simulated Annealings and Differential Evolutions), and this gap remains in this field. Especially, matheuristics are a new research field for Combinatorial Optimization problems. These methods are hybridizations between a heuristic approach and a mathematical programming model, where the mathematical programming model is a tool within the heuristic (FISCHETTI and FISCHETTI, 2018).

As discussed before, new features on SMPs have been investigated in recent years. Hence, multi-objective approaches on SMPs associated with assignment repairperson and complex systems (*k*-out-of-*n*) should be studied because excellent insights in decision-making processes were achieved when these aspects were separately addressed in SMPs. So, this joint approach could result in promising outcomes through metaheuristic and matheuristic resolutions.

The new problem proposed here can model many real applications, such as water distribution networks (due to head loss, should exist constraints on the minimal number of working pumps), processing plants (reliability restrictions of the management force the system to have a minimal number of working components), multi-display on cockpit systems, nuclear power plants (KANG and KIM, 2012), and others. In conclusion, there are some gaps to be fulfilled by this work: a lack in SMP that tackles multi-objective, repairperson assignment and complex systems together; exact, metaheuristic and matheuristic algorithm investigations and comparisons; and modelling of new real situations through the proposed problem.

1. 2 OBJECTIVES

The general objective of this dissertation is to model the bi-Objective Selective Maintenance and Repairperson Assignment Problem on *k*-out-of-*n* Systems (bi-OSMRAP:*k*-out-of-*n*).

About the specific objectives, one can list:

- To review the literature about SMPs, considering problem features and resolution methods;
- To propose the new bi-OSMRAP:*k*-out-of-*n*;
- To propose new metaheuristic and matheuristic algorithms for the problem resolution, and;
- To compare exact, metaheuristic and matheuristic algorithms results.

1. 3 DISSERTATION STRUCTURE

The structure of this dissertation is the following: the next chapter presents the theoretical background for this research and the literature review on SMP; chapter three discusses the methodology used in this work, showing the bi-OSMRAP:*k*-out-of-*n* mathematical model, and the full enumeration, metaheuristic and matheuristic algorithms; the subsequent chapter shows the computational results and their proper discussions; and finally, the last chapter states the dissertation's conclusions.

2 THEORETICAL BACKGROUND AND LITERATURE REVIEW

This chapter brings the theoretical background used in this research, besides the literature review about the SMP, and they are separated into two sub-chapters for the best organization. The first sub-chapter discusses some definitions about systems, components, action features, mission features and multi-objective terms, whereas the other shows the literature review about works on the SMP. These works were categorized according to their characteristics.

2. 1 THEORETICAL BACKGROUND

For our understanding, De Jonge and Scarf (2020) present a good review of asset and maintenance management terminology. Therefore, most of the terms presented here were obtained from their work. So, *system* is an asset that performs an operational function, and it is the physical goal of the maintenance management. Systems can be dismembered into minor parts, e. g. sub-systems, units, and go on. These *components* or *units* are parts of the system subject to maintenance actions, and they are the most essential part (cannot be subdivided). *Single-unit* or *single-component* systems are composed of only one component, differently from *multi-unit* or *multi-component* systems. The deterioration process of components is governed by transitions between states from the *deterioration state space*. Here we are dealing with just two-state spaces: *working* or *failed*.

Concerning actions, they can be classified into *preventive* (*programmed*) and *corrective*. The preventive ones are performed before the component failure, whereas correctives are executed after. *Repairs* and *replacements* are maintenance actions to be performed in components, and *non-repairable components* are maintained only by replacement. Unlike non-repairable components, repairs for *repairable components* can be *perfect*, returning the component to a state *as-good-as-new*, or *imperfect*, otherwise. The *minimal repair* is a special kind of imperfect action, which restores the component to its status prior to failure (*as-bad-as-old*).

About mission features, according to Ciappa (2005), when an item must fulfill a specific task during a defined time and under certain conditions, this job is called *mission*. The time between two sequential missions is called *maintenance break* (CAO et al., 2018), and the system probability of successfully fulfilling a specific mission with time duration t is called *system reliability for a mission of duration time t*. Regarding the disposition of components, multi-component systems can be arranged into *series*, *parallel*, *series-parallel*, k-out-of-n, among others. Specifically, k-out-of-n systems are the ones where the system works when at least k out of n components work (YAVUZ et al., 2019). On the other hand, *Repairpersons* are the people that compose the crew responsible for maintenance actions executions. Finally, *virtual* and *effective ages* are synonymous, and they describe an essential feature in maintenance modeling. Although the actual age of the component is different, its functional features behave as a new component with that effective age.

According to Keller (2017), multi-objective optimization is divided into *generating* and *preference-based methods*. The first one looks for the Pareto optimality without considering the decision-maker's preferences, while the other integrates the decision-maker in its resolution. Jaimes et al. (2010) indicate that multi-objective problem resolutions are based on *Pareto dominance*. Furthermore, Hwang and Masud (1979) establish that a solution is *optimal* when simultaneously optimizing each objective. However, as most multi-objective problems have *conflicting objectives*, there is no optimal solution. So, *Pareto-optimal* or *nondominated solutions* are the ones in which no one objective can be improved without a simultaneous detriment to at least one of the other objectives of the problem. The set composed of all Pareto-optimal solutions is called *Pareto Frontier*.

Ideal and *Nadir vectors* are essential notions in multi-objective optimization. Miettinen (1999) states that *ideal vectors* are the ones in which each objective is optimized, but they are not feasible. However, they are essential references, behaving as "bounds" for the Pareto optimal set. *Nadir vectors* can be feasible or not, and they are composed of the worst objective values over the Pareto set. While Nadir points are defined on the Pareto set, ideal points are defined over the search space.

2. 2 LITERATURE REVIEW

The SMP was initially defined by Rice (1998) on series-parallel subsystems composed of independent and identical components governed by exponential lifetime distributions. Only corrective actions are allowed, such that failed components should be replaced to maximize the system's reliability to the next mission. Extending this work, Cassady et al. (2001) also consider independent and identical components, but now governed by Weibull lifetime distributions. In addition, actions to be performed can be minimal repairs, replacements and corrective actions. To the best of our knowledge, they present the first exact procedure (a full enumeration), and this exact algorithm was incorporated into a Monte Carlo simulation for a

25 subsequent mission solving. Iyoob et al. (2006) solve SMP on series-parallel systems with identical components and constant failure rates through a modified enumeration procedure from the redundancy allocation problem. They consider resource buyings, which lead to cost and reliability improvements, and subsequent missions are resolved through a Markov chain modeling.

Since then, many new features have been considered in SMPs. We listed two review papers indicating recent advances in this field. Xu et al. (2015) listed 70 works between 1998 and 2014, classifying them according to problem characteristics and solution procedures. Cao et al. (2018) do a systematic review on SMP, and they classify the works from system configurations to working conditions. In addition, they pointed out solution methodologies used in SMPs (full enumerations, Genetic Algorithms, Simulated Annealing, Tabu Search, Branch and Bound algorithm, for instance). Finally, they propose a framework for Selective Maintenance Optimization with four phases and conclude their work with a presentation of SMP shortcomings on functional dependencies, stochastic models, working conditions considerations and scheduled maintenance actions.

2.2.1 SMP on fleet-level

When the SMP addresses separated and identical systems responsible for fulfilling identical missions, then the problem is defined at fleet-level. Schneider and Cassady (2004) modeled SMP in this context with constant failure rates on components. The problem solving is obtained by mixing a full enumeration and simulation, where decisions between two missions are made through the full enumeration. Furthermore, the same authors study this SMP with cost optimization and abortion mission possibility (SCHNEIDER and CASSADY, 2015). Expressly, when a mission is aborted (for all fleet components), then a penalty cost incurs. They present three problems, and their solutions are compared for a set of instances randomly created.

Different from these problems, bi-OSMRAP:k-out-of-n is devoted to only one system, but extending it to tackle multiple similar systems is a not too hard task, like Schneider and Cassady (2004) did, extending the original SMP.

2.2.2 Exact algorithms for SMP

Because SMP is a non-linear problem, enumerations are pretty much the only exact procedure for solving large instances. So, Rajagopalan and Cassady (2006) created an efficient solution methodology for large instances of SMPs with series-parallel systems and subsystems with identical components. Four improvements were proposed and compared with the complete enumeration for 30000 instances. These strategies are based on establishing limits for the variables and objectives for computational effort reduction. For some instances, the computational effort was reduced by 99%, but the proposed methods are sensitive to the number of subsystems.

Lust et al. (2009) compared heuristics/metaheuristics with exact procedures, and the model addressed by the authors is equivalent to a non-linear and non-separable Knapsack Problem. They propose a non-greedy heuristic based on the selection of actions with the highest rate between the reliability increase and the time duration of the maintenance action. On the other hand, the exact algorithm is based on the Branch-and-Bound (B&B) methodology with time relaxations. In this implicit enumeration algorithm, its initial solution is generated by the proposed non-greedy heuristic, which is also used in the Tabu Search algorithm. For the tests, they considered 13 instances with a number of components between 4 and 28. The full enumeration becomes inefficient for systems with more than 20 components, while the algorithm based on B&B reaches all optimal solutions for the tested instances. Tabu Search had a mean relative error of 0.21%, with computational times less than 1s, and the authors conclude that the metaheuristic is a fair competitor to the exact algorithm.

Galante and Passannanti (2009) modeled SMP on series-parallel systems subject to perfect replacements only. The proposed algorithm is based on a modification of Kettele's algorithm, which determines the number of redundancy components on subsystems for reliability maximization. They utilized two criteria for solution space reduction, based on B&B lower and upper limits. Finally, the algorithm was tested on a naval unit with 199 components, and its execution took a short time.

In a recent work, Cao et al. (2016) consider a series-parallel system with imperfect actions, and proposed an algorithm for solution space reduction. Firstly, one verifies whether the maintenance scheme that makes all components available is feasible. Otherwise, schemes are generated similarly to the B&B algorithm with depth-first-search. However, the procedure is susceptible to the budget limit.

Unlike bi-OSMRAP:*k*-out-of-*n*, these works are denoted for identical components (RAJAGOPALAN and CASSADY, 2006) or they do not consider imperfect actions (LUST et al., 2009; GALANTE and PASSANNANTI, 2009), and these exact algorithms cannot be directly applied to the new problem proposed here. On the other hand, B&B techniques are used in all algorithms, demonstrating to be an efficient tool to solve these problems, but Branch-and-Cut algorithms also should be studied, for example.

2.2.3 Imperfect action models

Imperfect actions put the component in a state between *as-good-as-new* and *as-bad-as-old*. This change is commonly described by effective age and/or hazard failure rates. Two particular models are the age reduction coefficient of Malik (1979) and the hybrid hazard rate of Lin et al. (2000). Both are used in many works, as Liu and Huang (2010), and these authors were the first to deal with SMP under imperfect action utilizing Malik (1979)'s model. Their work uses a Genetic Algorithm for the problem of coal transportation in a power station that supplies a boiler. On the other hand, Pandey et al. (2013a) extend Lin et al. (2000)'s work, proposing a hybrid model of age reduction and hazard adjust. A Differential Evolution algorithm was used for SMP resolution, and they concluded that imperfect actions considerations are crucial for the reliability increase.

2.2.4 SMP and repairperson assignment

The output from the SMP solving is the set of maintenance actions to be performed. These actions should be executed by a maintenance crew, logically, and the dissociation of the decisions about actions and repairpersons generates sub-optimal solutions. So, Khatab et al. (2018) were the first to study the SMP and repairperson assignment, achieving interesting conclusions: how higher is the inferior reliability limit, more repairpersons are hired, and more components have actions to be performed in them; how lower is the budget, fewer repairpersons are hired; when the budget permits, high and medium-level skilled repairpersons are hired, otherwise medium and low-level are hired; and usually mix teams are more beneficial than homogeneous.

Considering sustainable objectives, Khatab et al. (2019) extend Khatab et al. (2018)'s problem using new and remanufactured spare parts. Specifically, component lifetimes are dictated by statistically different lifetimes from the sub-populations. They conclude that remanufactured spare parts seem very beneficial and can offer financial advantages with minimal budget availability.

Chaabane et al. (2020) consider multimissions, and they minimize total costs subject to a minimal reliability between missions. They propose an Elitist Genetic Algorithm for the problem solution and conclude that as the minimal limit of reliability decreases, fewer repairpersons are hired, decreasing costs. Additionally, as this minimal limit increases, more pro and regular repairpersons are hired otherwise, trainees and regular ones are hired. Mixed teams reached better solutions regarding costs, reliability and computational complexity. Finally, as the reliability inferior limit increases, more component replacements occur, and highly skilled repairpersons are required.

SMP for fleet-level systems and the repairperson assignment was discussed by Khatab et al. (2020). They transform the original problem into a binary problem by listing all pattern schemes (combinations of maintenance actions, component states and repairpersons selected). After this transformation, they solve the equivalent Multidimensional Knapsack Problem. Imperfect actions avoid mission postponements and additional budget requirements, and the algorithm reaches optimal solutions in 180 seconds for large problems.

Khatab et al. (2018) and Khatab et al. (2019) works are different from ours on the *k*-out-of-*n* and multi-objective considerations. On the other hand, this work here do not study the environmental advantage to use remanufactured parts. Additionally, Chaabane et al. (2020) extend the work of Khatab et al. (2018) to consider multimissions instead only one, like in our work here.

2.2.5 SMP with stochastic parameters

Quddoos et al. (2015) pointed out that bi-objective fuzzy SMP models are needed due to difficulties in parameter estimations. They treat the number of failed components, available budget and action costs as trapezoidal fuzzy numbers, and the stochastic problem is transformed for a deterministic solving. Extending this work, Diallo et al. (2019) propose a bi-objective binary programming model with cost and reliability goals, and they resolve it with the weight sum method. They conclude that how higher is the importance given to the reliability, more repairpersons are used. Additionally, for large values of the constraint of minimal system reliability, the weight given to the reliability almost does not impact the optimal solution.

Certa et al. (2011) model the SMP with optimization of time and maintenance costs, subject to a minimal system reliability. An exact approach is created for Pareto Frontier generation through equivalence and dominance criteria usage, and an instance with 199 components in series-parallel disposition was tested. Another work in this context is of Das Adhikary et al. (2016) that utilized the NSGA-II for availability and cost optimizations. Zhao and Zeng (2017) tackle the SMP where break duration times are exponentially distributed through a multi-mission approach. They solve it through an algorithm of hybrid intelligent optimization based on empirical rules.

Some interesting works model the SMP with stochastic variables (KHATAB et al., 2016; KHATAB et al., 2017a; KHATAB et al., 2017b). Khatab et al. (2016) evaluate imperfect actions considering their quality factors as random variables. Illustrative examples conclude that neglecting the stochasticity of the quality of actions leads to errors in the policy evaluation. The deterministic model does not have feasible solutions for one tested example, while the probabilistic proposed model produces a feasible maintenance plan.

Khatab et al. (2017a) treat break and mission duration times as random variables. Their objective is to minimize the total maintenance time subject to constraints of minimal system reliability and maximum budget. They conclude that without the stochasticity in models, there is an overestimation of the system reliability. Due to the random variables, actions can be interrupted in the breaks, so one variable was introduced to compute the proportion of activities executed during the breaks. They found out that to neglect the stochasticity yields a discrepancy of 4% in the reliability system. Finally, Khatab et al. (2017b) deal with the same problem, and the authors achieved that, for symmetric distributions for the duration time of missions, the system reliability is almost insensible to the stochasticity of the variables.

Summarizing, for some real situations, SMP parameters need to be considered as stochastic variables to minimize mistakes in modeling and maximize the robustness of the solution provided. Looking all previously cited works, different parameters are dealt as random variables, but in bi-OSMRAP:k-out-of-n the problem is deterministic.

2.2.6 Special SMPs

Two papers are very special for this research because they were unique to tackle k-out-of-n systems (DIALLO et al., 2018) and objectives distinct from the SMP literature (ZHANG et al., 2020). They provided some interesting insights for this work about mathematical formulations. So, Zhang et al. (2020) minimized the energy consumption, and they propose a reliable energy consumption model. A Gravitational Search Algorithm is used for the problem resolution. In addition, comparisons with other algorithms are stated, and the authors achieved an insight about the problem behavior: the energy consumption and the constraint of minimal reliability have a non-linear relation.

About complex systems, Diallo et al. (2018) model the SMP with *k*-out-of-*n* systems. A unique contribution of this paper is the exact approach for SMP resolution, which transforms the non-linear problem into a binary problem (a Multidimensional Knapsack Problem). The problem of non-linearity is eliminated by listing all patterns because their reliability computations are part of the pre-optimization process. Therefore, the problem becomes to select patterns that optimize the reliability, subject to resource constraints. Two non-linear models were given, one with cost minimization and another with reliability maximization. The exact approach reaches better results than the two discussed models, and the authors conclude that considering the imperfect actions permits interesting optimal solutions.

The proposed problem in this work can be considered as a straight extension of the Diallo et al. (2018)'s work, because the two problems are very similar, except by the multi-objective approach in bi-OSMRAP:*k*-out-of-*n*.

2.2.7 SMP and maintenance dependencies

Maaroufi et al. (2013) study economical and functional dependencies through global failure propagation and isolation effects. These effects provide isolation of the dependent components, preventing failures. Economical dependencies occur in the breaks between missions. The authors propose five rules for solution space reduction for large problems, and the illustrative case shows that these rules reduce the solution space by 75%. They use a simulation/mathematical model for problem solution providing.

In bi-OSMRAP:*k*-out-of-*n*, components are statistically independent, but there is a lot of practical situations where components impact to each other and the original model must be corrected.

2.2.8 SMP with multi-state systems

Most of the discussed works consider binary states for components, but it is not reasonable to make this assumption in some cases. Pandey et al. (2013b) focus on multi-state systems with different output rates. These performance levels are discretized, and the system degradation is governed by a homogeneous markovian model (continuous Markov chain), such that the problem solving is made through an Evolutionary Algorithm. It was conducted a sensitive analysis on the problem, and they found out some interesting conclusions: there is an interval for the budget increasing that generates interesting reliability improvements; imperfect actions considerations lead to the reliability increase; as more resources are available, the components and actions to be performed are changed; and resource assignments depend on component states and the system performance.

Dao and Zuo (2017) consider systems with a non-constant load condition governed by a Normal distribution. In other words, the system degradation rate depends on the component state and load. The SMP model has time and cost constraints, and a simulation approach solves it. Chen et al. (2012) address the SMP with one model for the load distribution among components in multi-state systems, maximizing the system reliability. Cao et al. (2017) state a fuzzy model with uncertainty in the multi-state components' performance capacity and state transition intensity.

Because components in bi-OSMRAP:*k*-out-of-*n* are assumed to be in one out of two states (operational or in failure), the problem do not consider multi-state systems. However, for train systems for example, the system can visit different states.

2.2.9 SMP with multi-horizon planning

We could cite two papers about the horizon planning for two or more subsequent missions. Pandey et al. (2016) applied the SMP over a finite horizon planning, where its goal is to determine the optimal number of intervals and preventive actions for failure and total cost minimizations. On the other hand, Maillart et al. (2009) tackle infinite and finite horizon multi-mission planning through stochastic programs. Binary components are subject to identical missions, and the problem program maximizes the expected total number of successful missions. Comparing two subsequent missions and single mission approaches, they found out that, for subsequent missions, it is optimal to "sacrifice" the reliability of the next mission to avoid a situation in which all components are failed. In addition, comparisons among myopic, two missions, *t* missions and infinite mission policies were made. For 1000 artificial instances, the myopic policy produces optimal solutions for all instances, except for 34 of them and, even when it is not optimal, it is nearly optimal.

To sum up, SMP literature has been well studied, presenting many new problems and algorithms. However, we can see some shortcomings, such as few works modeling component dependencies and other important objectives, like environmental objectives. On the other hand, repairperson assignments seem to be a solid issue in SMPs, and stochastic SMPs have been widely argued.

3 METHODOLOGY

This chapter presents the research featuring of this dissertation, besides the mathematical model for the bi-OSMRAP:*k*-out-of-*n*, and the full enumeration, metaheuristic and matheuristic algorithms.

3. 1 RESEARCH FEATURING

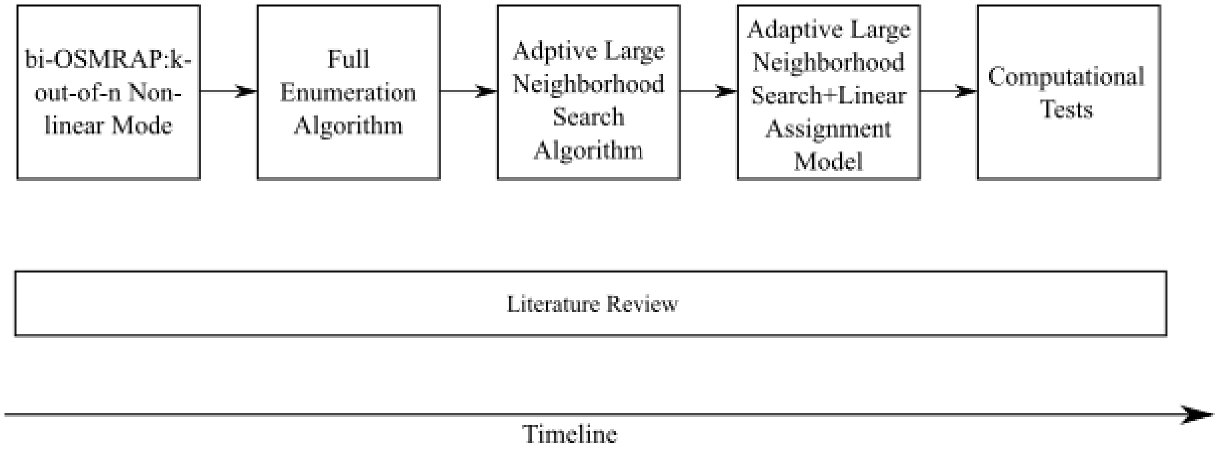
This research is featured as:

- Regarding nature, it is applied because it is focused on solving a specific problem;
- Regarding methodology, it is quantitative because we use analytical and approximated models to solve the problem;
- Concerning objectives, it is exploratory since it involves a bibliographic survey, and, descriptive because it describes a specific context, without the researcher interference;
- About the research procedure, it can be said experimental.

State-of-art on exact algorithms for mono-objective SMPs (DIALLO et al., 2018; KHATAB et al., 2020) are based on full enumerations, where the problem non-linearity is eliminated through SMP transformation into a Multidimensional Knapsack Problem, and this equivalent problem is solved through a linear resolution method. As mono and bi-objective optimizations have different output natures, it is not necessary a linear problem transformation because we are looking for the Pareto Frontier of the problem, which is obtained with non-dominance relations. So, we adapted this full enumeration idea for the Pareto Frontier generation of bi-OSMRAP:*k*-out-of-*n*. Since it is NP-hard, we also propose an *Adaptive Variable Neighborhood Search* (AVNS), and a matheuristic based on this algorithm and the linear repairperson assignment model.

To summarize, this research methodology is composed of the steps shown in Figure 1.

Figure 1- Dissertation methodology.



Source: The author (2022).

The literature review is conducted during the whole work because it is a continuous activity. The first task is the mathematical problem statement, and then we model the Full Enumeration Algorithm. So, metaheuristic and matheuristic are proposed, and computational tests are conducted for all algorithms.

3.2 MATHEMATICAL MODEL

Bi-OSMARP:*k*-out-of-*n* mathematical model is supported by some assumptions:

- **The system is composed of multiple, statistically independent and repairable binary components.** Statistically independent components is an assumption to neglect dependencies on components. Also, components assume two states - failed and operational. With this assumption, component reliability computations are made easier;
- **During the break, components do not age (age only is operation-dependent).** The break duration is not sufficient for failure mode arising from the environment, thus we consider only degradation from the operation;
- **During the mission, no maintenance activity is allowed other than minimal repair when a component is in failed state and this maintenance action has no effect on the failure rate.** Because SMP goal is the establishment of the action plan to be performed during breaks, action executions are only permitted in the break, except minimal repairs on components that failed during the mission;

- **Time to perform minimal repair during mission is neglectable.** For the same reason that previous assumption, time for minimal repair during mission is also neglectable. In other words, costs incurred after the break are not counted;
- **All resources are available when required.** Resource unavailability should be represented by some penalization in model parameters, which is not predicted in the model;
- **Multiple components can be worked on simultaneously without repairpersons colliding.** For many systems, e. g. multi-pump systems, production lines and navy vessels, their components can be accessed without interference on remainder components. Because they are large systems, different repairpersons can work on different components without hindering costs occurrences.

This work tackles multiple *k*-out-of-*n* subsystems in series (said *s* subsystems, each with s_i parallel components). There are *n* different and statistically independent components E_{ij} (i=1,..,s and j=1,..,s_i) where for each subsystem *i*, at least k_i components must be operational for the subsystem to be considered functional. The system just finished a mission and it is in preparation for the next one. That is, maintenance activities will be carried out during the next Ψ time units (break time) and the next mission lasts *U* time units. For each component, variables B_{ij} and A_{ij} denote the effective ages of E_{ij} components at the beginning and end of the break, respectively. Furthermore, X_{ij} and Y_{ij} dictate the functioning state of component E_{ij} at the beginning and end of the break, respectively (1 if E_{ij} is functioning and 0 otherwise). Component effective ages are perfectly known at the beginning and end of the break.

Concerning actions, preventive and corrective actions can be performed on system's components. Each failed E_{ij} has a list of action options to be executed, composed by L_{ij}+1 actions $\{0,1,...,l,...,L_{ij}\}$. Actions 0, 1, *l* and L_{ij} represent "Do nothing", minimal repair (it brings the component to an *as bad as old* state), an intermediate action (imperfect action) and *as good as new* maintenance action, respectively. All imperfect actions lead to a quality improvement of the component, generally given by age reduction and/or hazard failure rates decreasing. So, utilizing the age reduction proposed by Malik (1979), corrective maintenance level *l* reduces the component age by $\alpha_{ijl}\backslash 0\leq\alpha_{ijl}\leq 1$ and takes t_{ijl}^{c} time units. Similarly, preventive maintenance level *l* is given by $l\in\{0,2,...,L_{ij}\}$. Actions from 2 to L_{ij} rejuvenate the component age through reduction by a factor $\delta_{ijl}\backslash 0\leq\delta_{ijl}\leq 1$ and their performing last t_{ijl}^{p} time units.

Maintenance crew is responsible for action executions. So, there are r repairpersons (r=1,...,m), which are associated parameters c_r^f and c_r^v to them, denoting fixed and variable costs, respectively. Variable costs directly depend on the spent time by repairpersons.

Therefore, the decisions are about the action to be performed for each E_{ij} and the repairperson responsible to do it. Binary variables y_{ijlr} and z_r indicate these decisions. When y_{ijlr}=1, then maintenance level l is performed on component E_{ij} by repairperson r, and variables z_r that indicate the repairperson hiring or not.

Now, from the effective age of component E_{ij} immediately before the break starting (B_{ij}), its initial status (X_{ij}) and maintenance level practiced (l) we can define the effective age of component E_{ij} after the break time (A_{ij}) through Equation (1).

$$A_{ij} = B_{ij}\left[X_{ij}\sum_{r=1}^{m}\sum_{l=0,l\neq 2}^{L_{ij}}\delta_{ijl}.y_{ijlr} + (1-X_{ij})\sum_{r=1}^{m}\sum_{l=0}^{L_{ij}}\alpha_{ijl}.y_{ijlr}\right]. \tag{1}$$

Note that the two terms in Equation (1) are mutually exclusive because each one describes preventive and corrective maintenance in a dissociated way.

The probability of the system surviving the next mission is denoted by R. It depends on each component's reliability and, naturally, individual aggregation for each subsystem. The reliability of component E_{ij} for the next mission, when it is in working state at the ending of the break, is $R_{ij}^c(U|_{Aij})$. This conditional probability depends on mission duration (U) and the effective age at the beginning of the mission (A_{ij}). Applying Bayes' theorem, $R_{ij}^c(U|_{A_{ij}}) = \frac{R_{ij}(A_{ij}+U)}{R_{ij}(A_{ij})}$, where $R_{ij}(t)$ is the unconditional probability of surviving a mission which lasts t time units. Here we are considering lifetime distributions dictated by Weibull distributions with shape and scale parameters β_{ij} and η_{ij}, respectively. That is, $R_{ij}(t) = exp\left(-\left(\frac{t}{\eta_{ij}}\right)^{\beta_{ij}}\right)$. Therefore, the reliabilities for each subsystem (R_i) and the whole system (R) are:

$$R_i = \sum_{j_{k_i}=1}^{s_i}\sum_{j_{k_i-1}=1}^{j_{k_i}-1}\cdots\sum_{j_1=1}^{j_2-1}\left(\prod_{v=j_1}^{j_{k_i}}R_{iv}^c\right)\left(\prod_{u=1,u\neq j1,\dots,j_{k_i}}^{j_{k_i}}(1-R_{iu}^c).Y_{ij}\right), \tag{2}$$

$$R = \prod_{i=1}^{s}R_i^s = \prod_{i=1}^{s}\left[\sum_{j_{k_i}=1}^{s_i}\sum_{j_{k_i-1}=1}^{j_{k_i}-1}\cdots\sum_{j_1=1}^{j_2-1}\left(\prod_{v=j_1}^{j_{k_i}}R_{iv}^c\right)\left(\prod_{u=1,u\neq j1,\dots,j_{k_i}}^{j_{k_i}}(1-R_{iu}^c).Y_{ij}\right)\right]. \tag{3}$$

This reliability computation is not straight because it requires a series of computations with mutually exclusive events, i. e. a full enumeration (ARULMOZHI, 2002). For instance, the reliability to survive the next mission for an 1-out-of-3 subsystem (parallel subsystem) is the sum of probabilities of: all three components may be functioning, any two of three components may be functioning and remaining one failed and any one of three components may be functioning and remaining two failed. Taking the last event as an example and assuming Rel_i the surviving reliability of component i, then this event is described by: Rel_1.(1-Rel_2).(1-Rel_3)+Rel_2(1-Rel_1).(1-Rel_3)+Rel_3(1-Rel_1).(1-Rel_2). All possibilities of one component to be operational and the other two failed are modeled in this equation.

On the other hand, costs involved in maintenance actions executions are let by C, which is denoted by:

$$C = \sum_{i=1}^{s}\sum_{j=1}^{s_i}\sum_{r=1}^{m}\left[c_r^f . z_r + (1 - X_{ij})\sum_{l=0}^{L_{ij}} c_r^v . t_{ijl}^c . y_{ijlr} + X_{ij}\sum_{l=0,l\neq 1}^{L_{ij}} c_r^v . t_{ijl}^p . y_{ijlr}\right], \tag{4}$$

where each term represents, in sequence, hiring costs, corrective costs and preventive costs. Similarly, the time spent on these actions for each repairperson r is given by T_r:

$$T_r = \sum_{i=1}^{s}\sum_{j=1}^{s_i}\left[(1 - X_{ij})\sum_{l=0}^{L_{ij}} t_{ijl}^c . y_{ijlr} + X_{ij}\sum_{l=0,l\neq 1}^{L_{ij}} t_{ijl}^p . y_{ijlr}\right]. \tag{5}$$

The non-linear binary program that models bi-OSMRAP:k-out-of-n is indicated below.

$$Max\, R = \prod_{i=1}^{s}\left[\sum_{j_{k_i}=1}^{s_i}\sum_{j_{k_i-1}=1}^{j_{k_i}-1}\cdots\sum_{j_1=1}^{j_2-1}\left(\prod_{v=j_1}^{j_{k_i}} R_{iv}^c\right)\left(\prod_{u=1,u\neq j1,\ldots,j_{k_i}}^{j_{k_i}} (1 - R_{iu}^c)\right). Y_{ij}\right] \tag{6}$$

$$Min\, C = \sum_{i=1}^{s}\sum_{j=1}^{s_i}\sum_{r=1}^{m}\left[c_r^f . z_r + (1 - X_{ij})\sum_{l=0}^{L_{ij}} c_r^v . t_{ijl}^c . y_{ijlr} + X_{ij}\sum_{l=0,l\neq 1}^{L_{ij}} c_r^v . t_{ijl}^p . y_{ijlr}\right] \tag{7}$$

Subject to:

$$\sum_{j=1}^{s_i} Y_{ij} \geq k_i, \forall i \in \{1, \ldots, s\} \tag{8}$$

$$\sum_{i=1}^{s}\sum_{j=1}^{s_i}\left[(1 - X_{ij})\sum_{l=0}^{L_{ij}} t_{ijl}^c . y_{ijlr} + X_{ij}\sum_{l=0,l\neq 1}^{L_{ij}} t_{ijl}^p . y_{ijlr}\right] \leq \Psi z_r, \forall\, r \in \{1, \ldots, m\} \tag{9}$$

$$\sum_{i=1}^{s}\sum_{j=1}^{s_i}\left[(1 - X_{ij})\sum_{l=1}^{L_{ij}} y_{ijlr} + X_{ij}\sum_{l=2}^{L_{ij}} y_{ijlr}\right] \geq z_r, \forall\, r\ \in \{1, \ldots, m\} \tag{10}$$

$$\sum_{r=1}^{m}\left[\sum_{l=0}^{L_{ij}} (1 - X_{ij}). y_{ijlr} + \sum_{l=0,l\neq 1}^{L_{ij}} X_{ij}. y_{ijlr}\right] = 1\,, \forall\, i\ \in\ \{1, \ldots, s\}\ and\ j \in \{1, \ldots, s_i\} \tag{11}$$

$$y_{ij1r} \leq 1 - X_{ij}, \forall\, i \in \{1,\dots,s\}, j \in \{1,\dots,s_i\}\; and\; r \in \{1,\dots,m\} \tag{12}$$

$$Y_{ij} = X_{ij} + \sum_{r=1}^{m}\sum_{l=1}^{L_{ij}} (1 - X_{ij}).\,y_{ijlr}\,.\,z_r, \forall\, i \in \{1,\dots,s\}\; and\; j \in \{1,\dots,s_i\} \tag{13}$$

$$A_{ij} = B_{ij}\left[X_{ij}\sum_{r=1}^{m}\sum_{l=0,l\neq 1}^{L_{ij}} \delta_{ijl}.\,y_{ijlr} + (1 - X_{ij})\sum_{r=1}^{m}\sum_{l=0}^{L_{ij}} \alpha_{ijl}.\,y_{ijlr}\right], \forall\, i \in \{1,\dots,s\}\; and\; j \in \{1,\dots,s_i\} \tag{14}$$

$$R_{ij}^{c}(U|_{A_{ij}}) = \frac{R_{ij}(A_{ij} + U)}{R_{ij}(A_{ij})}, \forall\, i \in \{1,\dots,s\}\; and\; j \in \{1,\dots,s_i\} \tag{15}$$

$$y_{ijlr}, z_r = \{0,1\}, 0 \leq R_{ij}^{c}(U|_{A_{ij}}) \leq 1, \forall\, i \in \{1,\dots,s\}, j \in \{1,\dots,s_i\}\; l \in \{1,\dots,L_{ij}\}\; and\; r \in \{1,\dots,m\}. \tag{16}$$

Equations (6) and (7) state the bi-objective problem feature, which maximizes the system reliability and minimizes maintenance actions costs. Constraints (8) indicates the *k*-out-of-*n* feature for each subsystem. Equations (9) force hired repairperson *r* to perform actions without exceeding the break duration time and Equations (10) that he/she is hired before any action execution, excluding "Do-nothing" and minimal repairs for working components. Restrictions (11) guarantee that only one maintenance action can be carried out on component E_{ij}. Equations (12) establish that minimal repairs are possible only on failed components, while Constraints (13), (14) and (15) update the working states, effective ages of components and compute conditional probabilities, respectively. Finally, restrictions (16) state natural constraints on decision variables (binary decisions) and the interval of conditional probabilities.

In conclusion, bi-OSMRAP:*k*-out-of-*n* maximizes the system reliability while minimizing maintenance actions costs for a set of *k*-out-of-*n* subsystems connected in a series way. There is a set of repairpersons capable of executing the maintenance actions with different performances and costs. Specifically, its objective is to find the Pareto Frontier (PF) by selecting the: set of components to be maintained, set of maintenance levels to be practiced on each selected component, number of repairpersons to be hired and assignments these repairpersons to each maintenance level.

3. 3 FULL ENUMERATION ALGORITHM

Figure 2 shows the pseudo-code for exact Pareto Frontier generation for bi-OSMRAP:*k*-out-of-*n* through full enumeration (FEA). This algorithm is presented in the work of Lima et al. (2021).

Figure 2- Pseudo-code of the Full Enumeration Algorithm for bi-OSMRAP:*k*-out-of-*n*.

Algorithm 1 Full Enumeration Algorithm

1: Input data: Instance data (n, s, s_i U, Ψ, k_i, η_{ij}, β_{ij}, X_{ij}, B_{ij}, r, c_r^f, c_r^p, L_{ij}, α_{ijl}, δ_{ijl}, t_{ijl}^c and t_{ijl}^p)
2: -Create a list (ζ_i) with all possible combinations of maintenance actions, such that at least k_i components will be working.
3: **for** all j $\in \zeta_i$ **do**
4: -Compute the reliability (R_j).
5: -Create a list (π_{ij}) with all feasible combinations of repairperson assignments for solution j, such that time constraints are satisfied (Equation 3.(9)).
6: **for** all k $\in \pi_{ij}$ **do**
7: -Compute the cost (C_{ijk}).
8: -Store this solution in Π, $\Pi_{ij}|\Pi_{ij} = \zeta_i \cup \pi_{ij}$, where Π is the list with all feasible solutions.
9: **end for**
10: **end for**
11: -Initialize a list (NDS) which will contain all non-dominated solutions
12: **for** all i $\in \Pi$ **do**
13: **for** j< $|NDS|$ **do**
14: **if** ($R_i \leq R_j$ and $C_i > C_j$) or ($R_i < R_j$ and $C_i \geq C_j$) **then**
15: -Solution i is dominated by j.
16: **else if** ($R_i \geq R_j$ and $C_i < C_j$) or ($R_i > R_j$ and $C_i \leq C_j$) **then**
17: -Solution i dominates j, therefore solution j is taked out from NDS list.
18: **end if**
19: **if** Solution i is not dominated by any solution in NDS **then**
20: -Store solution i in NDS
21: **end if**
22: **end for**
23: **end for**

Source: The author (2022).

In pseudo-code above, we generated all feasible combinations of maintenance actions, such that *k*-out-of-*n* constraints are satisfied (line 2). Because subsystem reliabilities depend only on performed actions (it does not matter which repairperson will perform the action), these reliabilities can be computed without the repairperson assignments (line 4). Then, all possible repairperson assignments are generated for every maintenance combination, meeting the time restrictions in Equation (9) (line 5). Now, cost computations can be realized (line 7), and non-dominance relations are investigated. So, a list of the current non-dominated solutions is initialized (line 11), and for each solution (combination of maintenance action and repairperson assignment), we compare it to every solution in the non-dominated solution list. This solution is discarded (line 15), takes out one solution in *NDS* (line 17) or is put into the list (line 20).

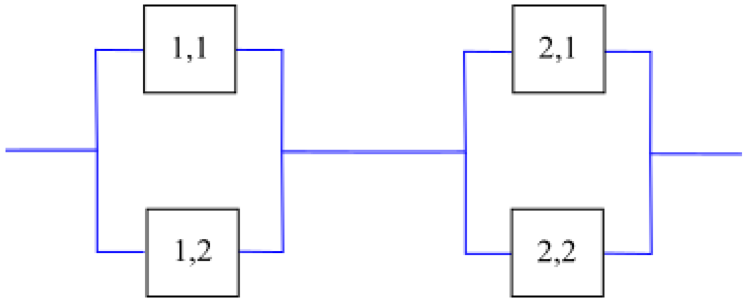
If we consider that all components have the same list of actions ($L = L_{ij}, \forall\, i \in \{1, \ldots, s\}\ and\ j \in \{1, \ldots, s_i\}$) and the worst instance case, i. e. when k-out-of-n restrictions are relaxed ($k_i = 0, \forall\, i \in \{1, \ldots, s\}$), all components are failed ($X_i = 0, \forall\, i\ \in \{1, \ldots, s\}$) and the break time is too long compared to maintenance actions, then there are $\sum_{i=0}^{n} m^i . (|L| - 1)^i . \binom{n}{i}$ possible solutions. In this equation, i indicates the number of components without "Do-nothing" actions assigned to them. For example, *i*=0 describes the case where all components have maintenance actions different from "Do-nothing" assigned to them. As actions different from "Do-nothing" will be performed on *i* components, then there are $(|L| - 1)^i$ possible actions to be performed on all components. Because each one of the *i* components could perform $|L| - 1$ actions, for all *i* components, there is $\binom{n}{i}$ possibilities of action combinations on the components. Finally, for each action different from "Do-nothing", all repairpersons can be assigned, and therefore it is multiplied by m^i.

This previous equation for the number of possible solutions is more sensitive to the number of components than maintenance levels and repairpersons. It has *n*-th exponential dependency and a combination of *n* factorial dependencies. However, on the computational view, a simple combination plays an exponential complexity in the worst case (when *i=n/2*). Therefore the algorithm has an exponential dependence on *n* in worst-case because the remaining variables have minor computational impacts.

3. 4 METAHEURISTIC ALGORITHM

This sub-chapter discusses the metaheuristic algorithm development, which is based on the *Multi-Start* and *Adaptive Variable Neighborhood Search* (AVNS). The solution representation is given by a *n*-th dimensional two-array, where one of them says which maintenance level should be practiced in each component, while the other states which repairperson is responsible for executing the action on the corresponding component. For example, a generic solution for the bi-OSMRAP:k-out-of-n of the system in Figure 3 is represented in Figure 4.

Figure 3- Fictitious problem.



Source: The author (2022).

Figure 4- Solution representation.

Components	1,1	1,2	2,1	2,2
Actions	0	1	3	1
Repairpersons	-	1	1	2

Source: The author (2022).

In Figure 3, a "Do-nothing" action is assigned to component 1 from subsystem 1 (1,1) and, therefore, none repairperson is assigned to this component ("-" in the array of repairpersons). Additionally, repairperson 1 is responsible for performing maintenance levels 1 and 3 on components 1,2 and 2,1, respectively, and repairperson 2 executes maintenance level 1 on component 2,2.

3.4.1 Initial Solution Construction

Searches in AVNS are performed from an initial solution using destroying and repair operators. So, we used a rustic Multi-Start method to created the initial PF. The number of iterations ($\Delta_{i,met}$), a greedy factor ($\tau_{i,met}$) and the probability of finishing the solution construction when it is already feasible (ρ) are input parameters. Figure 5 shows the pseudo-code for Multi-Start procedure.

Figure 5- Multi-Start algorithm for Initial Pareto Frontier.

Algorithm 2 Initial Pareto Frontier Creation

```
1:  Input data: Instance data ($n$, $s$, $s_i$ $U$, $\Psi$, $k_i$, $\eta_{ij}$, $\beta_{ij}$, $X_{ij}$, $B_{ij}$, $r$, $c_r^f$, $c_r^v$, $L_{ij}$, $\alpha_{ijl}$, $\delta_{ijl}$, $t_{ijl}^c$ and $t_{ijl}^p$) and algorithm parameters ($\Delta_{i,met}$, $\tau_{i,met}$, and $\rho_{met}$).
2:  for y∈ $\Delta_{i,met}$ do
3:      for i∈ $n$ do
4:          for j ∈ $L_{ij}$ do
5:              for k ∈ $r$ do
6:                  -Calculate the mean cost ($c$) among all possible repairpersons to be assigned to action j.
7:                  -Calculate the probability for component i through $(\sum_{i\in n} R_{ij}^c(U|_{A_{ij}}, j)/c.|L_{ij}|)^{\tau_{i,met}}$, without considering minimal repair on working components.
8:              end for
9:          end for
10:     end for                         ▷ Calculating components' selection probabilities
11:     for i∈ $s$ do
12:         j=0
13:         while j≤ $k_i$ do
14:             -Selects one component using the probabilities from line 6.
15:             -Generate a list ($\Lambda$) with all combinations of maintenance levels, excluding "Do-nothing" actions, and repairpersons.
16:             for x ∈ $\Lambda$ do
17:                 -Calculate the selection probability for x through $(R_{comp}^c(U|_{A_{comp},x})/c_{comp,x,r})^{\tau_{i,met}}$, where $c_{comp}$ is the cost associated to the combination of maintenance level and repairperson assignment for the component chosen.
18:             end for
19:             j+=1
20:         end while
21:     end for                         ▷ Selecting actions and repairpersons
22:     for i∈ $s$ do
23:         -RN=Random number between 0 and 1.
24:         if RN< $\rho_{met}$ then
25:             -Selects one action and repaiperson using the procedure from lines 10-20, but now considering "Do-nothing" actions. "Do-nothing" probabilities are just $R_{comp}^c(U|_{A_{comp},x})^{\tau_{i,met}}$.
26:         else
27:             -Finish this subsystem construction and go to the next.
28:         end if
29:     end for
30:     -Compute the solution reliability($R$).
31:     -Compute the solution cost($C$).
32: end for
33: - Perform non-dominance relations to create the PF.
```

Source: The author (2022).

Because we have two objectives to optimize, probabilities in line 7 from Figure 5 consider reliabilities and costs. Actions and repairpersons are selected for each subsystem to make it *k*-out-of-*n* feasible (lines 11-21). So with certain probability, new actions and repairpersons are added to the solution (line 25). Otherwise, the solution construction for this subsystem is finished and then the next subsystem is taken (line 27). "Do-nothing" actions are allowed in lines 22-29 because the solution already is *k*-out-of-*n* feasible. Finally, we compute the reliability and cost for the solution and non-dominance relations are performed (line 33).

3.4.2 Adaptive Variable Neighborhood Search Algorithm

Variable Neighborhood Search (VNS) is a metaheuristic proposed by Mladenović and Hansen (1997). VNS's framework consists of the successive application of neighborhood structures and one local search until there is no improvement in the incumbent solution. That is, there is a predetermined sequence of applying of the neighborhood structures, and this chain of structures is constantly repeated until there is no improvement in the current solution.

Many variants have been proposed for *Variable Neighborhood Search*. One of them, the *Adaptive Variable Neighborhood Search* (ANVS), has been studied in some multiobjective problems, like Facility Layout Problem (RIPON et al., 2013), Node Placement (ABDELKHALEK et al., 2015), Reentrant Flow Shop Scheduling (RIFAI et al., 2016). AVNS operates through the selection of one neighborhood structure, among many, and the selection probability is dynamically changed based on the neighborhood operator performance during the algorithm execution.

On the other hand, Das (1999) states that for multiobjective problems, the decision-makers usually pick one solution from the PF that "bulges out the most". It can be named as the point that achieves maximum "simultaneous improvement on the multiple objectives" and, therefore, has the highest value in terms of the marginal rate of the objective. So, during algorithm execution, we need to focus more on that region for the best exploration.

Operators in our metaheuristic are used to destroy and repair one solution from the PF, trying to find new good solutions. So, when one destroying operator is used, it removes an action/repairperson from the solution, and this solution is ”fixed” by a repair operator. Figure 6 shows the scheme for the AVNS, where for every iteration, a new single solution might be found through the using of two out of six operators for actions or repairpersons. Operators for action and repairpersons are very similar. From the six operators for each variable (actions and repairpersons), three of them are devoted to removing and the other half to repair, totaling twelve neighborhood operators.

Each one of these three kinds of operators have different rules for decision: greedy, pseudo-random or completely random. Specifically, for the greedy decision, the operator selects the action or repairperson with the lowest rate between reliability and cost. The pseudo-random operator computes probabilities through the rate between reliability and cost and power them to a greedy factor to take a pseudo-randomly decision. Finally, the last kind of operator takes a random feasible decision. For repairperson operators we just use the cost instead the rate between the objectives, and when "Do-nothing" actions are considered, then we calculate the rate with the current reliability of the component and a mean cost defined.

Figure 6- AVNS operation.

Algorithm 3 AVNS Operation

1: Input data: Algorithm parameters ($\Delta_{o,met}$, $\tau_{o,met}$, μ_{met}, $\lambda_{init,met}$, $\lambda_{end,met}$, $\kappa_{init,met}$ and $\kappa_{end,met}$).
2: -Initialize $\omega_i = 1, \forall\, i = 1, \ldots, 12$.
3: **while** i< $\Delta_{i,met}$ **do**
4: -Calculate the distances of all current PF solutions from the *Ideal Point*.
5: -From these distances, calculate the selection probability of each solution, powering it to $[\kappa_{init,met}, \kappa_{end,met}]$ according to the number of executed iterations.
6: -Selects one solution with these probabilities.
7: -Selects a number of removals for current iteration, based on μ_{met}. The probabilities are given by a Poisson distribution and depend to the number of executed iterations, where the mean is equal $\lambda_{init,met}$ at the beginning and $\lambda_{end,met}$ at end.
8: **for** j= $1, \ldots, \mu_{met}$ **do**
9: -Select one operator with probability depending on ω_is and remove a part of the solution.
10: -Rebuild the solution using repair operators.
11: **end for**
12: -Local Search()
13: -Based on the improvement in hypervolume, increase or decrease the ω_is.
14: **end while**

Source: The author (2022).

In pseudo-code above, input data are the number of iterations ($\Delta_{o,met}$), the greedy factor for pseudo-random decisions ($\tau_{o,met}$), the maximum number of action or repairperson removals in one iteration (μ_{met}), the mean for Poisson distribution for the number of removals at the beginning and end ($\lambda_{init,met}$ and $\lambda_{end,met}$), and the powering factor to focus on knee region of the PF ($\kappa_{init,met}$ and $\kappa_{end,met}$). Parameters (*λ's* and *κ's*) are linearized according to the number of iterations executed. Line 2 initializes the weights for each operator, making them equal to one. These weights carry the historical success of each operator, and operators with better historical are most likely chosen.

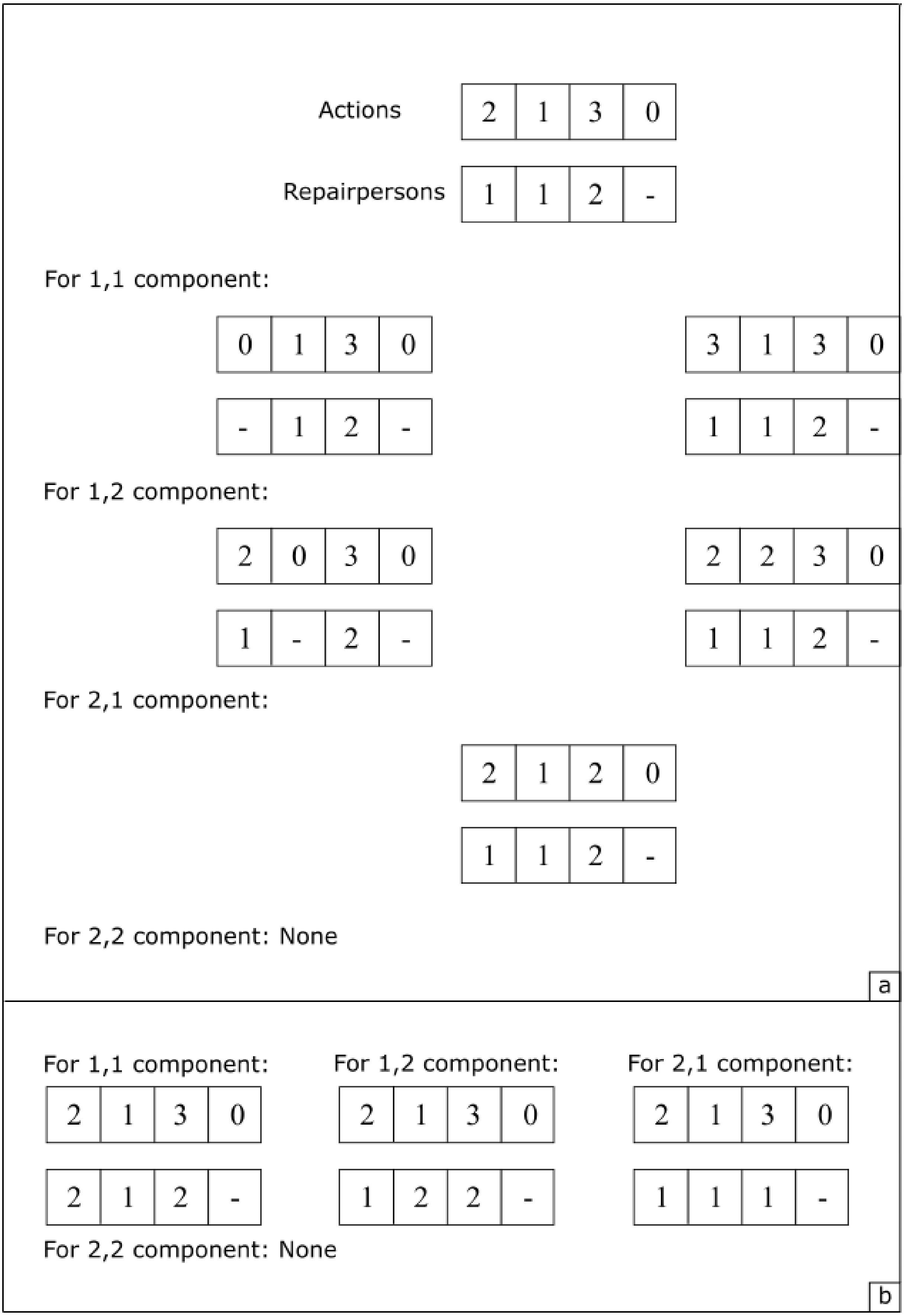
In every iteration of the AVNS, one solution from the current PF is taken and it is changed by the operators to get a new better solution. As previously said, the knee region of the PF is important for decision-makers. Therefore, to focus on this area, we develop a rule to explore solutions from this region at the algorithm ending, whereas at the beginning, searches in whole solution space are encouraged. The rule is following: we calculate selecting probabilities for each solution belonging to the approximated PF, and these probabilities are computed according to the distance from the *Ideal Point* (line 5). This point is composed of perfect objective values, named 100% of reliability and 0 cost. To focus dynamically on the knee region, we power the selecting probabilities to a greedy parameter. This parameter should have low and high values at the algorithm beginning ($\kappa_{init,met}$) and ending ($\kappa_{end,met}$), respectively. Finally, the powering parameter is linearized between these two values for the remaining iterations.

Nevertheless, selecting one solution from the knee region is not sufficient to guarantee that the searches will focus on this area. As more changes in solution are made (operators used), there is a natural trend to visit more distant regions. Therefore, we also proposed a rule to prevent many changes in the solution at the algorithm beginning and hence, the exploring of distant areas from the knee region. Thus, the number of removals is governed by a Poisson distribution with a mean between [$\lambda_{init,met}$, $\lambda_{end,met}$]. We discretized the number of removals from 2 to μ_{met}, such that $2 \leq \mu_{met} \leq n$. So, for every iteration, probabilities of removing 2, 3, . . . or μ_{met} are computed by the Poisson distribution. The mean is linearized between the initial and ending limits (line 7).

Action and repairperson operators are selected according to their weights. Destroying operators can remove "Do-nothing" actions too, but in this case, when we are rebuilding the solution, we must decide about the action and repairperson jointly. Thus, all feasible combinations of actions and repairpersons are considered.

Line 12 performs a local search with the new solution obtained. The local search is applied for each variable (actions and repairpersons) in different ways. Figure 3 shows the procedure when we considered the example from Figure 3 with 2 repairpersons and 4 maintenance levels. Components 1,2 and 2,2 are failed, while 1,1 and 1,2 are in working states.

Figure 7- Local search.



Source: The author (2022).

In Figure 7, the local search is divided into two, for actions (a) and repairpersons (b). For actions, two subsequent actions are investigated for each component since the original action is not a "Do-nothing" action. Using component 1,1 as an example, "Do-nothing" and level 3 actions were analyzed because component 1,1 is operational. Concerning repairpersons, each component with an action different from "Do-nothing" has all repairpersons analyzed. All components are analyzed in the local search for each decision, and every solution obtained from the local search is compared to the current approximated PF.

Multi-objective solutions are challenging to analyze because we cannot unify their behavior into one metric, like mono-objective problems. For example, Laszczyk and Myszkowski (2019) extensively review quality metrics for multi-objective problems. One of the most popular metric is the hypervolume, which is a metric for convergence, uniformity, and spread that computes the volume of the hypercube defined by the PF and *Nadir Point*. Therefore, the operators' weights used are updated based on the improvement, or not, of the hypervolume (line 13). The updating rule is: if the hypervolume does not change, the weights are decreased in one unit for each operator according to how many times it was used. If the hypervolume increased, the weights are increased according to the size of the change, always rounding up. For instance, if there were a 2.8% increase in hypervolume, we increase the weights in 3 units for each used operator how many times it was used.

3. 5 MATHEURISTIC ALGORITHM

Matheuristic algorithm is the same as the previous discussed, but now, every solution in the approximated PF is guaranteed to be cost-optimal. In other words, a linear program for repairperson assignment is solved for each new solution in the current PF. This model starts from a *k*-out-of-*n* feasible solution with actions assigned to the components. The cost optimal repairperson assignment is achieved through solving the following model:

$$Min\ C = \sum_{i=1}^{m} z_i c_i^f + \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} c_{ij}^v \tag{17}$$

Subject to:

$$\sum_{i=1}^{m} x_{ij} = WC_j\,, \forall j \in \{1, \ldots, n\} \tag{18}$$

$$\sum_{j=1}^{n} t_{ij} x_{ij} \leq \Psi z_r, \forall\ i \in \{1, \ldots, m\} \tag{19}$$

$$x_{ij} = \{0,1\}, z_i = \{0,1\}, \forall\, i \in \{1, \dots, m\}, j \in \{1, \dots, n\}. \quad (20)$$

In the model, z variables are the same as those from bi-OSMRAP:k-out-of-n. Variables x indicate repairperson i executes the action of component j if its value is 1. c_i^f's are fixed costs of repairperson hiring, like in bi-OSMRAP:k-out-of-n, and c_{ij}^v's are costs of repairperson i to execute the action assigned to component j. WC_j are binary values that indicate if component j has an action different from "Do-nothing", i. e. if WC_j=1, then there is an action different from "Do-nothing" in component j. t_{ij} are parameters that describe the time for repairperson i to perform the action in component j. Case WR_j=0, then t_{ij}=0 for all $i \in r$. Logically, c_{ij}^v's, WC_j's and t_{ij}'s depend on the set of actions selected for each component.

The objective function minimizes the costs for that solution. Constraints (18) indicate that when an action is different from "Do-nothing", one repairperson should be assigned to that action. Otherwise, no repairperson is assigned. Restrictions (19) impose that each repairperson must not exceed the break intermission time, and these restrictions are similar to (9) in bi-OMSRAP:k-out-of-n model.

So, the linear repairperson assignment model above is executed for each approximated solution in the PF. Therefore, the PF is guaranteed to be cost-optimal at the end of the algorithm execution. It is important to note that the model above is the same that models the *Single-Source Capacitated Facility Location Problem*.

4 COMPUTATIONAL RESULTS AND DISCUSSIONS

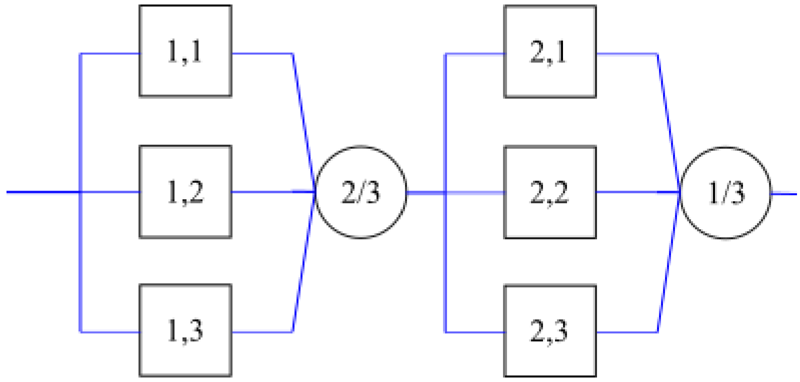
This chapter discusses the numerical experiments conducted, and we divided it into four sub-chapters. The first one (4.1) discusses an illustrative example of the new proposed problem. The second one (4.2) brings a sensitive analysis of the illustrative example. We show how the new problem behaves regarding changes in break intermission time, repairperson crew composition, imperfect action existence, quality parameters of the maintenance levels, the length of the next mission, *k*-out-of-*n* subsystems, lifetime distribution and cost parameters. The third sub-chapter (4.3) presents the metaheuristic and matheuristic resolutions of the illustrative example, besides the performance evaluation of the approximated PFs provided and the exact PF. Finally, the last one (4.4) resolves a variation of Liu and Huang (2010)'s instance, which is a moderately large instance and we can see the algorithm performances for a real size instance.

We evaluated the problem behavior on a small instance because we can know the exact PF through the FEA. Then, we compared the approximated PFs generated by the proposed algorithms to the exact PF. On the other hand, the PF providing for a moderately large instance requires a big computational effort. All computational tests were executed in a personal computer with an Intel i5 1.7GHz processor with 8GB RAM and 64bits operational system. All algorithms were implemented in Python language, specifically in IDE Spyder, while the repairperson assignment model in 3.3 was solved with Cplex.

4. 1 ILLUSTRATIVE EXAMPLE

We created an illustrative example to analyze the problem features. The system has 2 subsystems with 3 components each (Figure 8).

Figure 8- Illustrative example.



Source: The author (2022).

They are 2-out-of-3 and 1-out-of-3 subsystems, respectively. The second subsystem acts as a parallel system because the whole subsystem works when at least one component is operational. The break time (Ψ) is equal to 15 time units, while the next mission length (U) is 50. Table 1 shows the parameters for each component.

Table 1- Component parameters for the illustrative example.

Components	η	β	X	B
E_{11}	150	1.5	0	60
E_{12}	228	2.4	1	70
E_{13}	168	1.6	0	50
E_{21}	240	2.6	0	65
E_{22}	168	1.8	0	80
E_{23}	204	2.4	0	40

Source: The author (2022).

η and B are given in time units. Concerning the repairperson crew, we consider three action executioners named as trainee, regular and expert. Their fixed and variable costs are in Table 2.

Table 2- Repairperson crew costs.

Repairperson	c^f	c^v
Trainee	6	2
Regular	13	4
Expert	25	8

Source: The author (2022).

We considered four maintenance levels for this illustrative example. In other words, only one imperfect action is considered (the remaining are "Do-nothing", minimal repair and perfect replacements). All components have the same list of actions and age reduction factors are in Table 3.

Table 3- Age reduction factors.

Maintenance Level	α	δ
“Do-nothing”	1	1
Minimal Repair	1	-
Imperfect Replacement	0.5	0.3
Perfect Replacement	0	0

Source: The author (2022).

Table 4 brings the time to perform actions, according to the nature of the action.

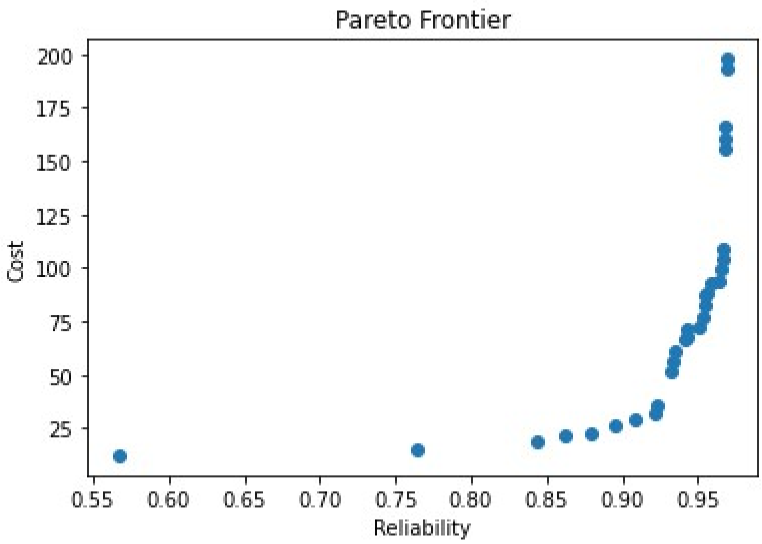
Table 4- Times to perform the maintenance actions for each repairperson.

Maintenance Level	t^c			t^p		
	Trainee	Regular	Expert	Trainee	Regular	Expert
“Do-nothing”	0	0	0	0	0	0
Minimal Repair	1.6	1.2	0.8	-	-	-
Imperfect Replacement	3.2	2.4	1.6	2.0	1.5	1.0
Perfect Replacement	10.4	7.8	5.2	4.4	3.3	2.2

Source: The author (2022).

Therefore, we run the illustrative example with the FEA from sub-chapter 3.2. Figure 9 shows the exact PF and the running time was 160 seconds.

Figure 9- Exact Pareto Frontier of illustrative example.



Source: The author (2022).

To understand the FEA complexity, we created 5 random instances with only one subsystem and Table 5 shows their features.

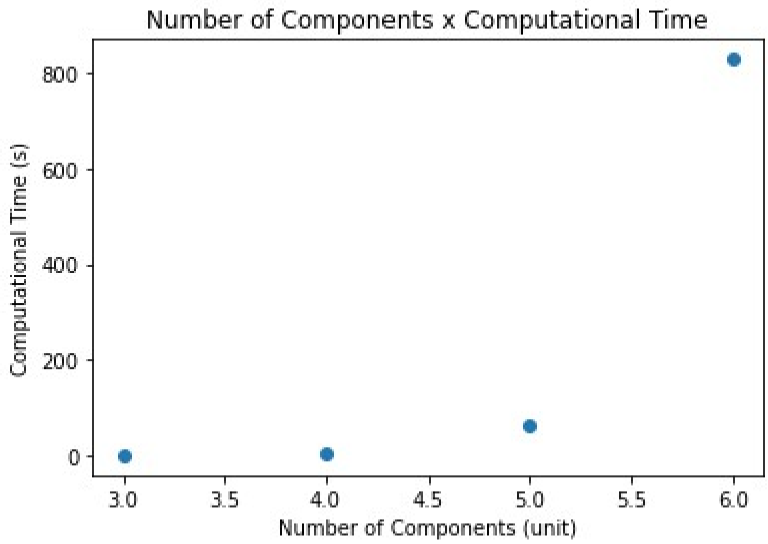
Table 5- Instance features for FEA performance evaluating.

Feature	Instance				
	1	**2**	**3**	**4**	**5**
n	3	4	5	6	7
$[\eta^{min}_{ij},\eta^{max}_{ij}]$	[15,20]	[15,20]	[150,240]	[150,240]	[150,240]
$[\beta^{min}_{ij},\beta^{max}_{ij}]$	[1.5,3]	[1.5,3]	[1.5,2.6]	[1.5,2.6]	[1.5,2.6]
B	[8,20]	[8,20]	[20,45]	[20,45]	[20,45]
k	2	3	3	4	4
$\sum_{i=1}^{n} X_i$	1	2	1	2	3
Ψ	8	19	15	15	15
U	10	10	50	50	50
$\|L\|$	3	4	4	4	3
$[t^{c,min}_{ij},t^{c,max}_{ij}]$	[0.9,9]	[0.7,9]	[0.8,10.4]	[0.8,10.4]	[0.8,10.4]
$[t^{p,min}_{ij},t^{p,max}_{ij}]$	[2,3]	[1.5,3]	[1.6,4.4]	[1.6,4.4]	[2.2,4.4]
m	2	3	3	3	3
$[c_r^{f,min},c_r^{f,max}]$	[50,60]	[50,70]	[6,25]	[6,25]	[6,25]
$[c_r^{v,min},c_r^{v,max}]$	[2,4]	[2,4,5]	[2,8]	[2,8]	[2,8]

Source: The author (2022).

In Table 5, the intervals indicate the amplitude of parameters in each instance. To evaluate the FEA difficulty to solve the instances, we gather the computational times for each instance into Figure 10. Instance 5 was run in 465.2 seconds representing a reduction of 43.92% compared to instance 4. Despite having more components, instance 5 is less difficult to resolve. For this context, imperfect action dismissing is more impacting than a marginal increase in *n*.

Figure 10- Computational times for the random instances.



Source: The author (2022).

Figure 10 has a substantial jump in execution time from instance 3 to 4. If we apply a potential regression fitting on Figure 10, we will obtain a correlation degree equal to 0.99875, and if we try to predict the computational time for an instance with 7 components, the expected time will be equal to 6246s or 1.735h. The required time will be far from acceptable for systems with more components, and therefore, approximated algorithms are necessary.

4. 2 SENSITIVE ANALYSIS

From the illustrative example, we introduce some single changes in parameters of the problem from 4.1 to understand the problem behavior. In subsequent figures, each legend and its meaning is showed in Table 6.

Table 6- Legend meanings.

Instance	**Meaning**
NM	The original instance
150%Ψ	50% increase in the break maintenance time
50%Ψ	50% decrease in the break maintenance time
TC	Homogeneous crew with trainee repairpersons
RC	Homogeneous crew with regular repairpersons
EC	Homogeneous crew with expert repairpersons
WIA	The original instance without the imperfect action
150%AF	50% increase in age factors for the imperfect action
50%AF	50% decrease in age factos for the imperfect action
U=10	Mission length equal to 10 units
k=1,1	1-out-of-3 and 1-out-of-3 subsystems (both parallel subsystems)
k=3,2	3-out-of-3 and 2-out-of-3 subsystems
k=3,3	3-out-of-3 and 3-out-of-3 subsystems (both series subsystems)
150%β	50% increase in shape parameters of the components' Weibull lifetime distribution
50%β	50% decrease in shape parameters of the components' Weibull lifetime distribution
150%η	50% increase in scale parameters of the components' Weibull lifetime distribution
50%η	50% decrease in scale parameters of the components' Weibull lifetime distribution
150%c^f	50% increase in fixed costs for the repairpersons
50%c^f	50% decrease in fixed costs for the repairpersons
150%c^v	50% increase in variable costs for the repairpersons
50%c^v	50% decrease in variable costs for the repairpersons

Source: The author (2022).

Firstly, changes in shape parameters are exciting because how lowest they are, lifetime distributions follow an exponential distribution, and how greater they are, the distribution acts as a normal distribution. On the other hand, changes in *k*-out-of-*n* restrictions are important because the nature of the subsystem is changed. For example, *k=1,1* expresses two subsystems with components in a parallel disposition, whereas *k=3,3* denotes a system with all components in series. Running times for all "new" instances are in Table 7.

Table 7- Running times for instances from the sensitive analysis.

Instance	Running time (s)
NM	160
150%Ψ	175
50%Ψ	123
TC	432
RC	583
EC	323
WIA	17
150%AF	163
50%AF	162
U=10	158
k=1,1	162
k=3,2	112
k=3,3	78
150%β	160
50%β	137
150%η	161
50%η	166
150%c^f	166
50%c^f	159
150%c^v	162
50%c^v	160

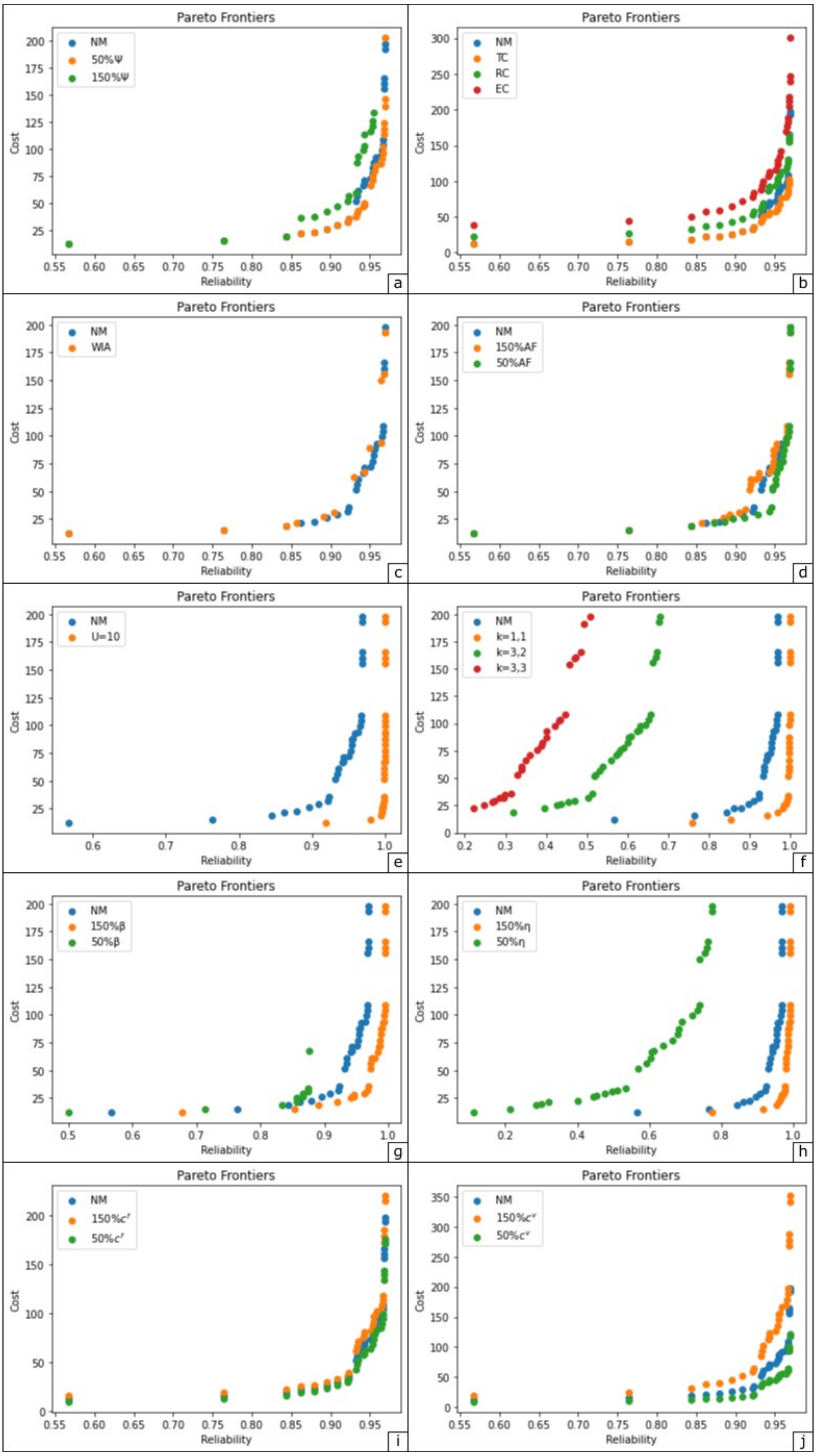
Source: The author (2022).

Comparing 150%Ψ and 50%Ψ with NM, there is an increase and decrease in running times, respectively. This behavior was expected because with more time to perform maintenance actions, solutions that were time unfeasible are now feasible, and more solutions are analyzed in FEA. Regarding homogeneous repairperson crews, all of them have increases in their execution times, meeting Chaabane et al. (2020) and Khatab et al. (2018) achievements, although they deal with different problems. Additionally, the imperfect action dismissing changes the execution time a lot, and as expected, changes in effective age factors and the next mission length do not significantly impact the execution time. On the other hand, we get some interesting conclusions about the changes in *k*-out-of-*n* constraints. With k_i's closer to s_i's (*k=3,3* and *k=3,2*), we observe an easiness to provide the solution. This easiness was expected because, in these cases, the reliability computations evaluate fewer events. On the other hand, we do not see significant impacts on running times for the changes in lifetime distribution parameters, except for 50%β, but there is no straight explanation for this. Finally, changes in fixed and variable costs do not impact the running times in a significant way.

Figure 11 shows the PFs for each "new" instance of the sensitive analysis. Because the evaluation of PFs is not a straightful task, we got some metrics from Laszczyk and Myszkowski (2019) to evaluate these solutions. We picked the hypervolume, Inverted Generational Distance (IGD), and ε metrics. IGD is an average distance from the points in true PF to the closest point in the approximated solution. On the other hand, ε measures the minimal distance required to move every point from the approximated solution so that every point in true PF is dominated. These indicators were originally designed to evaluate approximated PFs. However, all solutions are exact in this analysis, and we considered the PF of the original instance (NM) as the "true" PF and the instances from the sensitive analysis as "approximated". Finally, we calculated the percentage of the number of single solutions from the "approximated" PF that belong to the "true" PF (Ratio).

Metrics based on distances were computed with the normalized data using the *Nadir Point*. For fair analyses, for different cases of the sensitive analysis we considered different *Nadir Points*. So, Figures 11.a, 11.b, 11.c, 11.d and 11.e use the *Nadir Point* on 56.67% and 300.6 cost units. Figure 11.f uses 22.06% and 300.6, whereas 11.07% and 197.6 for 11.g and 11.h. Finally, the *Nadir Point* for Figures 11.i and 11.j was 56.67% and 351.2. Table 8 shows all indicators.

Figure 11- Sensitive analysis solutions.



Source: The author (2022).

Table 8- Metrics for each sensitive analysis solution.

Instance	Hypervolume	Ratio	Normalized IGD	Normalized ε
NM	112.97 / 214.58 / 153.80 / 131.16	-	-	-
150%Ψ	113.57	0.3000	0.01725	0.08186
50%Ψ	107.78	0.1000	0.05992	0.11214
TC	113.59	0.3000	0.06876	0.03938
RC	107.35	0.0000	0.03558	0.11032
EC	99.44	0.0000	0.05059	0.32648
WIA	111.61	0.2333	0.02036	0.02964
150%AF	112.24	0.8333	0.00498	0.01562
50%AF	114.06	0.6000	0.00753	0.02220
U=10	126.73	0.8666	0.06544	0.10019
k=1,1	229.10	0.1333	0.05188	0.08054
k=3,2	118.07	0.3333	0.32571	0.34971
k=3,3	63.69	0.0333	0.52010	0.51001
150%β	160.35	0.5667	0.03672	0.08778
50%β	139.78	0.0333	0.16517	0.06691
150%η	161.21	0.8000	0.04352	0.06830
50%η	100.21	0.5666	0.25986	0.45610
150%c^f	129.64	0.9000	0.01760	0.10018
50%c^f	132.69	1.0000	0.02145	0.11134
150%c^v	124.01	1.0000	0.02613	0.43736
50%c^v	134.74	0.9000	0.07096	0.06177

Source: The author (2022).

In Table 8, NM instance has different values for the hypervolume, as previously indicated, where the values are denoted to Figures 11.a, 11.b, 11.c, 11.d and 11.e; to 11.g; to 11.g and to 11.h; and to 11.i and 11.j, in sequence. Concerning the changes in break time and looking into Figure 11.a and Table 10, we can see that any 50% change in the break time significantly affects the solution. Specifically, the 50% decrease (50%Ψ) is six times more impacting than the 50% increase if we look the hypervolumes. We can also see this significant impact if we look at the ϵ metric because it presents a higher value to move the PF to make it equally non-dominated, and Ratio values are very low. To understand the IGD metric values, we need to have an idea of distances in the graphs. For example, the distance between the two subsequent solutions on the left tail of the original instance is equal to 0.19937. Therefore, IGD for 50% decrease have a significant value.

Regarding the impact of the crew composition (Figure 11.b), if we look the hypervolumes, we can note that cohort composed of experts decreases the hypervolume, showing a worsening of the PF quality. This quality deterioration is seen because experts are more expensive, but the PF for this case has a wide range of solutions, because with this kind of repairperson more top-level actions can be performed. For example, if we change expert's costs to make them equal to trainee's, then the hypervolume is equal to 117.37. Looking at the Ratio metrics, PFs are very different, but 30% of the TC solution is equal to NM, which indirectly indicates a majority use of trainees in NM. IGDs and ϵ's are not so high, indicating that the PFs are not so distant, except for EC. Therefore, since it is possible, the decision-maker should choose a homogeneous crew with trainee repairpersons instead of a heterogeneous with all kinds of repairpersons.

Figure 11.c and Figure 11.d analyze the impact of the imperfect action existence and the quality of this action. The imperfect action dismissing does not have a significant impact on the solution, as we can see in the hypervolume and looking at Figure 11.c. Further, if we observe Ratio, IGD and epsilon metrics, we see an evident similarity between WIA and NM solutions. The more significant difference between these the two solutions is the cardinality of the PFs because WIA has fewer single solutions. For changes in the quality of the imperfect action, one can note that the solutions are very similar, and these similarities can be seen with Ratio, IGD and epsilon metrics values and in a visual diagnostic of Figure 11.d.

Specifically for Figure 11.e, it is evident that a decrease in mission length will push the whole PF to the right because the same solutions will have higher reliabilities. However, from the point of view of the change in PF, we expected that the same single solutions belong to both PFs, but we noted a difference of 13.3%. Logically, the new solution will have better hypervolume and considerable IGD and epsilon metrics. In Figure 11.f we can see noticeable differences for k-out-of-n restrictions, and these changes provide some exciting insights. If we increase the k values in those restrictions, we need to have more operational components, and this should increase the system's reliability at the first moment, but this is not the behavior described by the figure. For example, taking a solution that belongs to NM and $k=3,3$, this solution satisfies both 3-out-of-3 restrictions in the two instances. Therefore, their costs are equal, but their reliabilities are not because the concept of the mission success was changed. Now, the subsystems are successful if all components survive until the finishing of the mission. This change in the concept of mission success is a significant difference. Therefore, when these constraints are tight, the reliability drops. Additionally, any change in this subsystem feature significantly impacts the PF composition, although PFs for NM and $k=1,1$ are similar, as indicated by low IGD and epsilon metrics values. These two metrics have high values for the remainder of the instances, indicating that both PFs are very distant.

When lifetime distributions are similar to an exponential function (50%β), the quality of the PF drops a lot, whereas that for normal distributions (150%β) there is an improvement (Figure 11.g). This behavior is expected because exponential distributions are too much disperse around the mean, while normal distributions are more centered. However, 50%β solution is not robust (low Ratio metrics), while 150%β has 40% of equal to solutions to those in NM. In other words, both 150%β and 50%β yield significant different PFs, justified by big IGD values. On the other hand, the problem is more robust regarding scale parameters changes (Figure 11.h). Analyzing the hypervolumes, the PF for 150%η is better than NM and 80% of the solutions are equal, while that for 50%η there is a significant deterioration of the PF. Concerning the solution composition of the PF, we see an interesting behavior. IGD and ϵ metrics are low for 150%η, supporting the similarity between the two PFs, whereas for 50%η they are greater, and the solutions are significantly different. In conclusion, if managers are reluctant about the scale parameters of the lifetime distributions and think that these parameters are superestimated, they should be concerned because the optimal PF should present different single solutions.

Finally, Figures 11.i and 11.j discuss the data for variations in cost parameters. For fixed and variable costs changes, all PFs are similar, presenting hypervolumes close to each other and low IGD and epsilon metrics values. Additionally, the problem is robust in relation to these cost parameters because with significant changes in their values (50% for more or less), at least 90% of the original PF remains optimal.

Also, we need to understand the crew composition for every solution in PFs and how these cohorts are modified when we introduced the changes on the sensitive analysis. Therefore, for each “new” problem created, we counted the percentile of each crew cohort composition when one and two repairpersons were used, besides the number of single solutions that used one (1Rep), two (2Rep) and all three repairpersons (OVNG), and these data are shown in Table 9.

Table 9- Percentiles of each cohort scheme.

Instance/Percentil	Trainee	Regular	Expert	Trainee and Regular	Trainee and Expert	Regular and Expert	1Rep	2Rep	OVNG
NM	100%	0%	0%	100%	0%	0%	9	16	30
150%Ψ	100%	0%	0%	100%	0%	0%	15	14	30
50%Ψ	100%	0%	0%	64%	36%	0%	3	11	19
WIA	100%	0%	0%	80%	20%	0%	6	5	13
150%AF	100%	0%	0%	100%	0%	0%	10	17	32
50%AF	100%	0%	0%	100%	0%	0%	10	17	31
U=10	100%	0%	0%	100%	0%	0%	10	15	30
k=1,1	100%	0%	0%	100%	0%	0%	10	11	26
k=3,2	100%	0%	0%	100%	0%	0%	8	18	31
k=3,3	100%	0%	0%	100%	0%	0%	8	14	28
150%β	100%	0%	0%	100%	0%	0%	10	16	31
50%β	100%	0%	0%	100%	0%	0%	9	1	10
150%η	100%	0%	0%	100%	0%	0%	11	16	32
50%η	100%	0%	0%	93%	7%	0%	12	14	31
150%c^f	100%	0%	0%	84%	16%	0%	9	19	30
50%c^f	100%	0%	0%	100%	0%	0%	9	16	30
150%c^v	100%	0%	0%	100%	0%	0%	9	16	30
50%c^v	100%	0%	0%	84%	16%	0%	9	19	30

Source: The author (2022).

Data from the second to fourth columns are denoted when one repairperson is hired, while from the fifth to seventh two repairpersons are used. In Table 9, we can see that trainees are used most of the time for almost all cases. When two of them are used, trainee and regular repairpersons are practically always used, except for some changes (50%Ψ, WIA, 50%η, 150%c^f and 50%c^v). This behavior is expected for the break time decrease, for example, because in this case, actions must be executed quickly, and the expert is faster than regular. On the other hand, for the imperfect action dismissing, without an intermediary action, perfect replacements are performed and they need to be quickle executed to leave time to perform other actions.

Table 9 indicates that trainees are mainly used. Therefore, it is interesting to define ranges for cost parameters to make the regular and expert preferable instead trainees. Thus, Table 10 shows some values for the costs and maintenance crew’s composition.

Table 10- Percentiles of each cohort scheme according to costs parameters.

Instance/Percentil	Trainee	Regular	Expert	Trainee and Regular	Trainee and Expert	Regular and Expert	1Rep	2Rep	OVNG
NM	100%	0%	0%	100%	0%	0%	9	16	30
c_2^f=1	78%	22%	0%	100%	0%	0%	9	16	30
c_3^f=6	100%	0%	0%	69%	31%	0%	9	16	30
c_2^f=1 and c_3^f=6	78%	22%	0%	100%	0%	0%	9	16	30
c_2^v=2	67%	33%	0%	100%	0%	0%	12	13	30
c_2^v=1	20%	80%	0%	100%	0%	0%	12	13	30
c_3^v=4	53%	0%	47%	30%	70%	0%	17	10	29
c_2^v=1 and c_3^v=4	17%	83%	0%	81%	0%	19%	12	16	30

Source: The author (2022).

In Table 12, we can see that even significant changes in fixed costs for regular repairperson (c_2^f=1) lead to just minor changes in the trainee usage when only one repairperson is used. On the other hand, the decrease in expert's fixed cost (c_3^f=6) provides no change in crew cohort when one repairperson is used, but when two of them are used, there is a substantial change. When both changes are made jointly, the result is similar to when c_2^f=1. Unlike fixed cost, variable costs changes presented more significant impacts. For example, when c_2^v=1, regular repairperson is mostly used, and when c_3^v=4, trainee and expert have practically the same utilization. Finally, doing c_2^v=1 and c_3^v=4 at the same time, the schemes change a lot in comparison with NM, and we see a majority use of the regular repairperson. In conclusion, decreasing the fixed cost for regular and expert repairpersons forces them to be hired. On the other hand, variable costs changes are much more impacting, not only hiring more repairpersons, but also changing the hierarchy of use of the repairpersons.

To sum up, we can conclude that the break time, crew composition, and lifetime distribution's parameters play the most significant impacts on PFs. On the other hand, the imperfect action's existence and quality do not. Additionally, it seems that exists an interval in which the gain from the marginal increase begins to be ineffective for the break time. In other words, there is a point in which the decision-maker should stop investing in the break time increase to maximize his/her gain. *K*-out-of-*n* constraints noticeably impact the PF, and they need to be delicately modeled as well as the lifetime distributions. Thus, if the decision-maker has available resources to increase the solution quality, he/she should allocate them in crew composition, break time (if it is not reached the clímax) and the exchange of the components by others with "better" lifetime distributions.

4. 3 METAHEURISTIC AND MATHEURISTIC PERFORMANCE EVALUATIONS

Now we will evaluate the metaheuristic and matheuristic performances from chapter 3 for the illustrative example.

Before the running of the algorithms, we need to set the parameters. So, the calibration process rule was: we took each parameter once and varied it in a reasonable interval, keeping the remaining constant. The value with the best behavior is set. Table 11 shows the parameter values.

Table 11- Algorithm parameters after the calibration process for the illustrative example.

Parameter	**Value**
$\Delta_{i,met}$	1000
$\tau_{i,met}$	2
ρ_{met}	0.7
$\Delta_{o,met}$	10000
$\tau_{o,met}$	0.5
μ_{met}	2
$\lambda_{init,met}$	0.5
$\lambda_{end,met}$	0.1
$\kappa_{init,met}$	0.05
$\kappa_{end,met}$	0.5
$\Delta_{i,math}$	1000
$\tau_{i,math}$	2
ρ_{math}	0.7
$\Delta_{o,math}$	2000
$\tau_{o,math}$	0.5
μ_{math}	2
$\lambda_{init,math}$	0.5
$\lambda_{end,math}$	0.1
$\kappa_{init,math}$	0.05
$\kappa_{end,math}$	0.5

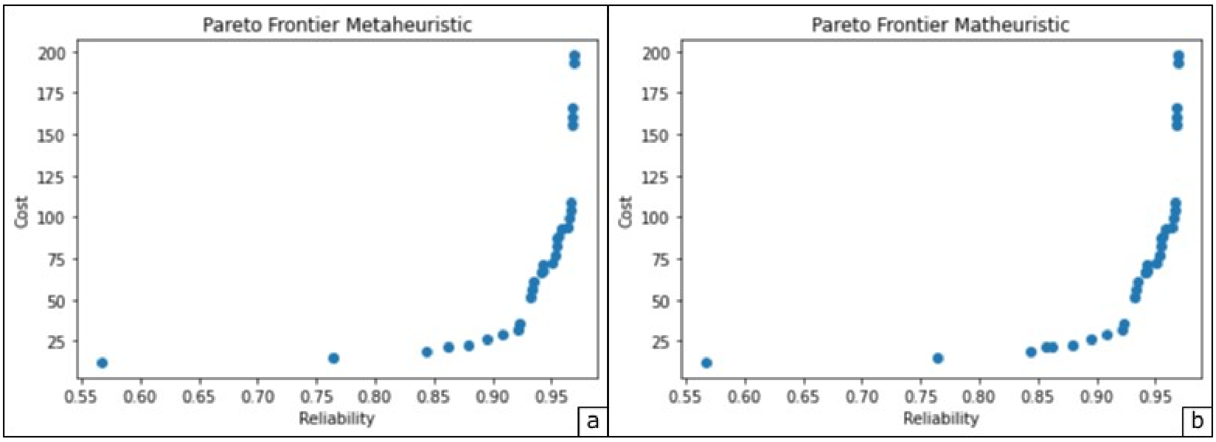
Source: The author (2022).

Looking into Table 11, practically all parameters, except the number of iterations, have the same values, and therefore, the following analyses are devoted for both. The initial solution greedy parameter is more than 1, indicating that for the initial PF creation, greedy decisions are beneficial. The same conclusion cannot be stated for AVNS searches because

τ_o's are 0.5, indicating that the best decisions are taken when non-greedy decisions are made. ρ's values indicate that additional components are selected before the solution finishing. The maximum number of action/repairperson removals in one iteration is 4, because μ′s=2. In other words, 4/12 of the solution is subject to change. Finally, about the dynamic parameters κ, they are less than 1, indicating that the searches are always diversified, giving more attention to solutions distant from the *Ideal Point*. Because the solution space is small, the algorithm searches in all PF regions without special attention on the knee region. Finally, λ's plays as expected, decreasing the changes at the ending.

To solve the illustrative example, we run each algorithm 30 times because they are stochastics, and examples of PFs generated are shown in Figure 12.

Figure 12- Examples of metaheuristic and matheuristic solutions for the illustrative example.



Source: The author (2022).

If we look into Figures 12 and 9, we can note that they are very similar. For the best evaluation, Table 12 shows the algorithm metrics for the 30 runs. We presented them on their average and standard deviations.

Table 12- Metaheuristic and matheuristic metrics for the illustrative example.

Metric	Metaheuristic	Matheuristic
Running Time (s)	39.62±1.14	59.81±31.05
Hypervolume	112.97±0.00	112.92±0.08
OVNG	30.00±0.00	32.37±2.28
Ratio	1.00±0.00	0.957±0.036
Normalized IGD	0.0000±0.0000	0.0049±0.0051
Normalized ε	0.0000±0.0000	0.0053±0.0108

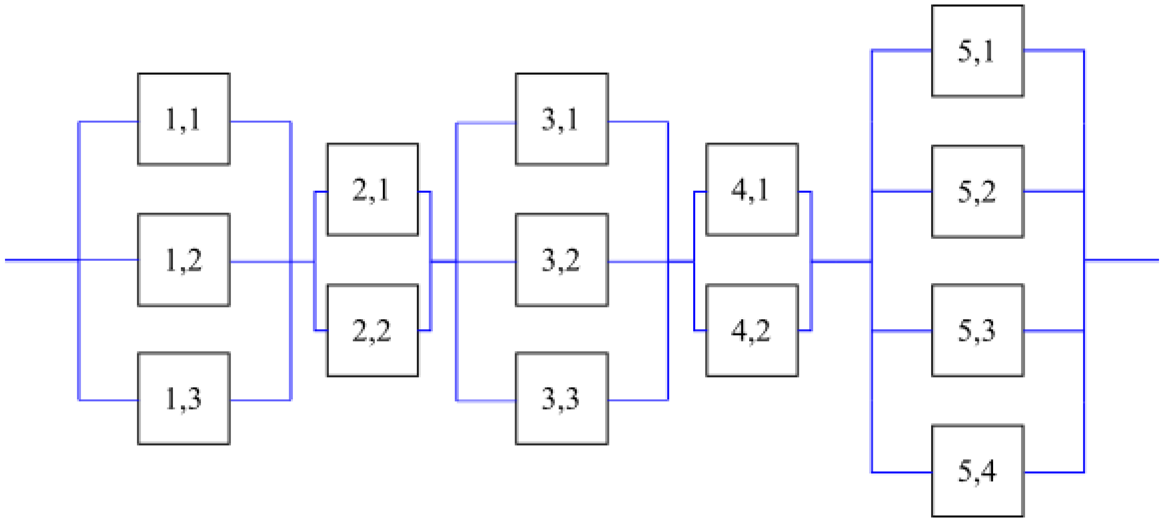
Source: The author (2022).

We see that matheuristic has high running times because of the linear models running for the repairperson assignments. Metaheuristic has an outstanding behavior achieving the true PF for all runs, while that matheuristic was not always perfect, but reached a very good behavior, finding 95.7% of the true PF in average. We suspect that the matheuristic was not so good as the metaheuristic because the metaheuristic evaluate many solutions in a minor computational time, while for each new solution of the matheuristic, the linear model is run, consuming much time. With the metrics, we can conclude that the algorithms provide excellent approximations in a reasonable time compared to the FEA. Still, we need to test the algorithms for a moderately large problem to validate their performances.

4. 4 LIU AND HUANG INSTANCE

To test metaheuristic and matheuristic algorithms performances, we solved the bi-OSMRAP:*k*-out-of-*n* on the coal transportation system in a plant supplying a boiler from Liu and Huang (2010), with 5 subsystems and 14 components in total. Because we are dealing with a new problem, the original Liu and Huang (2010)'s instance was modified to introduce the *k*-out-of-*n* features. System disposition for the original Liu and Huang (2010)'s instance is shown in Figure 13.

Figure 13- Liu and Huang (2010)'s instance.



Source: The author (2022).

Above, we considered that the subsystems are: 2-out-of-3, 1-out-of-2, 2-out-of-3, 2-out-of-2 and 2-out-of-4. Specifically, subsystems 2 and 4 operate as parallel and series subsystems, respectively. Table 13 shows the parameters of each component in this system.

Table 13- Component parameters of Liu and Huang (2010)'s instance.

Components	η	β	X	B
E_{11}	250	1.5	0	110
E_{12}	380	2.4	1	150
E_{13}	280	1.6	0	170
E_{21}	400	2.6	1	120
E_{22}	280	1.5	0	180
E_{31}	340	2.4	0	100
E_{32}	260	2.5	0	130
E_{33}	280	2.0	1	170
E_{41}	260	1.2	1	150
E_{42}	350	1.4	0	120
E_{51}	400	2.8	0	180
E_{52}	350	1.5	1	130
E_{53}	300	2.4	0	100
E_{54}	450	2.2	0	150

Source: The author (2022).

5 repairpersons were available for the repairperson crew, named 1 trainee, 2 regulars and 2 experts. Their fixed and variable costs are in Table 14.

Table 14- Repairperson crew costs for Liu and Huang (2010)'s instance.

Repairperson	c^f	c^v
Trainee	15	12
Regular	20	15
Expert	25	20

Source: The author (2022).

In the original instance were considered five maintenance levels, i. e., two kinds of imperfect actions are considered. All components have the same list of actions and the age reduction factors are in Table 15.

Table 15- Age reduction factors for Liu and Huang (2010)'s instance.

Maintenance Level	α	δ
"Do-nothing"	1	1
Minimal Repair	1	-
Imperfect Replacement 1	0.4	0.2
Imperfect Replacement 2	0.2	0.1
Perfect Replacement	0	0

Source: The author (2022).

Table 16 brings the time to perform actions, according to the nature of the action.

Table 16- Times to perform the maintenance actions for each repairperson for Liu and Huang (2010)'s instance.

Maintenance Level		t^c			t^p	
	Trainee	**Regular**	**Expert**	**Trainee**	**Regular**	**Expert**
"Do-nothing"	0	0	0	0	0	0
Minimal Repair	4	3	2	-	-	-
Imperfect Replacement 1	6	5	2.5	3	2.5	1.25
Imperfect Replacement 2	7	6	3	3.5	3	1.5
Perfect Replacement	8	7	4	4	3.5	2

Source: The author (2022).

The break time and next length mission are equals to 12 and 100 time units. The calibration process was executed for this instance, and the algorithm parameters are in Table 17.

Table 17- Algorithm parameters after the calibration process for Liu and Huang (2010)'s instance.

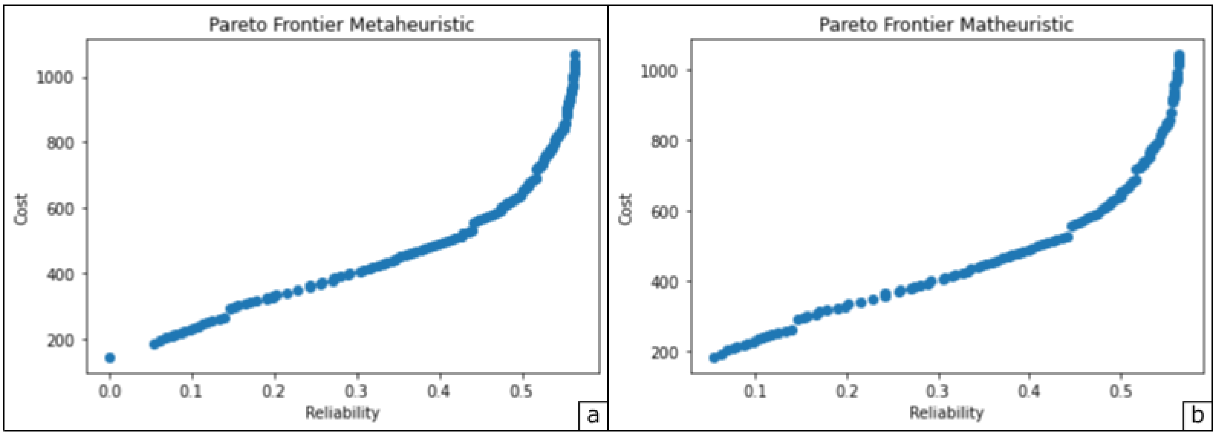
Parameter	**Value**
$\Delta_{i,met}$	3000
$\tau_{i,met}$	0.5
ρ_{met}	0.6
$\Delta_{o,met}$	15000
$\tau_{o,met}$	1
μ_{met}	2
$\lambda_{init,met}$	1.0
$\lambda_{end,met}$	0.5
$\kappa_{init,met}$	1.0
$\kappa_{end,met}$	2.0
$\Delta_{i,math}$	3000
$\tau_{i,math}$	2
ρ_{math}	0.8
$\Delta_{o,math}$	10000
$\tau_{o,math}$	1.0
μ_{math}	2
$\lambda_{init,math}$	1.0
$\lambda_{end,math}$	0.5
$\kappa_{init,math}$	1.0
$\kappa_{end,math}$	2.0

Source: The author (2022).

We can see an interesting behavior on initial greedy parameters. For the metaheuristic, non-greedy decisions are made, whereas the otherwise occurs in matheuristic. Besides that, the matheuristic begins with single solutions with more actions on components because ρ has a considerable value. Looking into μ, we can see that a few components are removed and repaired because just 4/28 of the solution is subjected to change. The κ's have the same values for both algorithms, and differently from the illustrative example, now the algorithms dynamically focus on the knee region. Finally, λ's describe a similar behavior to the illustrative example.

We run each algorithm 30 times, because they are stochastic algorithms. Examples of PF generated are shown in Figure 14.

Figure 14- Examples of metaheuristic and matheuristic solutions for Liu and Huang (2010)'s instance.



Source: The author (2022).

Table 18 shows the algorithm metrics for the 30 runs. Unlike the illustrative example, now we do not know the exact PF or the *Nadir Point*. So, we set this point with the worst values found in the preliminary tests (0.0% and 1100.0 cost units). Because we do not know the exact solution, IGD, epsilon and Ratio metrics used before cannot be used now, and we introduce the Hole Relative Size (HRS). This metric computes the biggest gap in the approximated solutions' distribution (LASZCZYK and MYSZKOWSKI, 2019). It takes the maximum spacing between two single solutions from the approximated PF and divides it by the average spacing between the two subsequent solutions.

Table 18- Metrics for metaheuristic and matheuristic performances on Liu and Huang (2010)'s instance.

Metric	**Metaheuristic**	**Matheuristic**
Running Time	187.82±4.06	456.52±44.98
Hypervolume	383.416±0.392	383.712±0.057
OVNG	141.31±4.72	132.931±5.18
HRS	5.7530±0.3173	5.7807±0.3250

Source: The author (2022).

Matheuristic running times are about 1.4 times slower than metaheuristic. Furthermore, its standard deviation is 3.6 times higher than for metaheuristic. Both algorithms achieve interesting values for hypervolumes, but the matheuristic is more robust than the metaheuristic. Metaheuristic has a better average and standard deviation in OVNG, but matheuristic also has a good behavior. Finally, HRS metrics are very similar and indicate a good output, with single solutions very close to each other.

Therefore, both algorithms achieve reasonable solutions, although the metaheuristisc is faster than matheuristic. This higher running time is the price paid to guarantee the cost optimality. However, metaheuristic solutions seem to reach this condition, even without a mathematical model inside it. In other words, the metaheuristic provides a fast PF with solutions that are equally good to those from matheuristic.

5 CONCLUSIONS

This dissertation dealt with the bi-Objective Selective Maintenance and Repairperson Assignment Problem on *k*-out-of-*n* systems (bi-OSMRAP:*k*-out-of-*n*), a new problem in Selective Maintenance field. This new problem addresses a multi-component system which will undergo maintenance actions for the preparation for the next mission. Different from other problems in Selective Maintenance, this one considers bi-objective optimization, multiple repairpersons and *k*-out-of-*n* subsystems. Additionally, real situations found in processing plants, display cockpit, multi-pumps systems and nuclear plants can be solved with this new problem.

To support the decision-maker, the mathematical non-linear model, exact, and two approximated algorithms were proposed. Both approximated algorithms are based on the *Adaptive Variable Neighborhood Search*. In addition, we proposed specific rules inside these algorithms to focus on the knee region of the Pareto Frontier. Two instances were resolved, an illustrative example and a modified instance from Liu and Huang (2010)'s work. Also, we did a sensitive analysis on some instance parameters for the first instance and validated the metaheuristic and matheuristic. Finally, we got some exciting achievements: to maximize the PF quality, the decision-maker should put their resources on the crew composition, break time increase, hiring more efficient repairpersons or replace components by new ones with "better" lifetime distributions. On the other hand, imperfect action existence and its quality do not improve the solution.

We can list some future works from this research, for example, the problem can also consider the environmental impact of the maintenance decisions. Furthermore, large and real instances should be modeled with bi-OSMRAP:*k*-out-of-*n*. In the computational results, we noticed an interval for decision-makers to pay for specific improvements in the number of resources. However, we need to extensively study this behavior to provide more insightful ideas. Also, it is common to see situations where there are economic agreements between suppliers and companies. In this case, some actions should have specific repairpersons to be assigned, or some repairpersons should have a minimal usage restriction. Furthermore, the study about the hard-constraint for the break time is necessary because associated gains with the increase in time to perform actions were exciting. Changing it to a soft-constraint with a penalty cost dynamics is an excellent way to understand this behavior. Last but not least, access dependencies on components should be studied too. For example, for some systems, components are accessed through the dismembering of the whole subsystem. In this case, actions on these hard-to-access components create an opportunity to reduce the cost in those components with easier access.

REFERENCES

ABDELKHALEK, O.; MASRI, H.; KRICHEN, S. An adaptive variable neighborhood search for solving the multi-objective node placement problem. *Electronic Notes in Discrete Mathematics*, v. 47, p. 189-196, 2015.

ARULMOZHI, G. Exact equation and an algorithm for reliability evaluation of *K*-out-of-*N*:*G* system. *Reliability Engineering and System Safety*, v. 78, p. 87-91, 2002.

BARLOW, R.; HUNTER, L. Optimum Preventive Maintenance Policies. *Operations Research*, v. 8, n. 1, p. 90-100, 1960.

BARLOW, R.; HUNTER, L.; PROSCHAN, F. Optimum checking procedures. *Journal of the Society for Industrial and Applied Mathematics*, v. 11, n. 4, p. 1078-1095, 1963.

BEVILACQUA, M.; BRAGLIA, M. The analytic hierarchy process applied to maintenance strategy selection. *Reliability Engineering and System Safety*, v. 70, n. 1, p. 71-83, 2000.

CAO, W.; JIA, X.; HU, Q. Selective maintenance optimizationfor multi-state systems subject to common cause failures. *Acta Aeronautica ET Astronautica Sinica*, v. 39, n. 2, DOI: 10.7527/S1000-6893.2017.221556, 2017.

CAO, W.; JIA, X.; HU, Q.; ZHAO, J.; WU, Y. A literature review on selective maintenance for multi-unit systems. *Quality and Reliability Engineering International*, v. 34, n. 5, p. 824-845, 2018.

CAO, W.; SONG, W.; HU, Q.; DU, Y. An Exact Method for Solving Selective Maintenance Problems Considering Imperfect Maintenance. In: International Conference on Intelligent Networking and Collaborative Systems, Ostrawva, p. 522-526, 2016.

CASSADY, C.; MURDOCK, W.; POHL, E. Selective maintenance for support equipment involving multiple maintenance actions. *European Journal of Operational Research*, v. 129, n. 2, p. 252-258, 2001.

CERTA, A.; GALANTE, G.; LUPO, T.; PASSANNANTI, G. Determination of Pareto frontier in multi-objective maintenance optimization. *Reliability Engineering and System Safety*, v. 96, n. 7, p. 861-867, 2011.

CHAABANE, K.; KHATAB, A.; DIALLO, C.; AGHEZZAF, E.; VENKATADRI, U. Integrated imperfect multimission selective maintenance and repairpersons assignment problem. *Reliability Engineering and System Safety*, v. 199, p. 106895, 2020.

CHEN, C.; LIU, Y.; HUANG, H. -Z. Optimal load distribution for multi-state systems under selective maintenance strategy. In: 2012 International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering, Chengdu, p. 436-442, 2012.

CIAPPA, M. Lifetime prediction on the base of mission profiles. *Microeletronics Reliability*, v. 45, n. 9-11, p. 1293-1298, 2005.

DAO, C.; ZUO, M. Optimal selective maintenance for multi-state systems in variable loading conditions. *Reliability Engineering and System Safety*, v. 166, p. 171-180, 2017.

DAS, I. On characterizing the "knee" of the Pareto curve based on Normal-Boundary Intersection. *Structural Optimization*, v. 18, p. 107-115, 1999.

DAS ADHIKARY, D.; BOSE, G.; JANA, D.; BOSE, D.; MITRA, S. Availability and cost-centered preventive maintenance scheduling of continuous operating series systems using multi-objective genetic algorithm: A case study. *Quality Engineering*, v. 28, n. 3, p. 352-357, 2016.

DE JONGE, B.; SCARF, P. A review on maintenance optimization. *European Journal of Operational Research*, v. 285, n. 3, p. 805-824, 2020.

DIALLO, C.; KHATAB, A.; VENKATADRI, U. Developing a bi-objective imperfect selective maintenance optimization model for multicomponent systems. *IFAC-PapersOnLine*, v. 52, n. 13, p. 1079-1084, 2019.

DIALLO, C.; VENKATADRI, U.; KHATAB, A.; LIU, Z. Optimal selective maintenance decisions for large serial *k-out-of-n: G* systems under imperfect maintenance. *Reliability Engineering and System Safety*, v. 22, p. 234-245, 2018.

FISCHETTI, M.; FISCHETTI, M. Matheuristics. In: MARTÍ, R.; PARDALOS, P.; RESENDE, M. ed. *Handbook of Heuristics*. Cham, Springer, 2018, cap. 5, p. 121-155.

GALANTE, G.; PASSANNANTI, G. An exact algorithm for preventive maintenance planning of series–parallel systems. *Reliability Engineering and System Safety*, v. 94, n. 10, p. 1517-1525, 2009.

HWANG, C. -L.; MASUD, A. Multiple Objective Decision Making - Methods and Applications. In: BECKMANN, M.; KUNZI, H. ed. *Lecture Notes in Economics and Mathematical Systems*. New York, Spinger-Verlag, 1979, cap. 1, p. 1-20.

IYOOB, I.; CASSADY, C.; POHL, E. Establishing Maintenance Resource Levels Using Selective Maintenance. *The Engineering Economist*, v. 51, n. 2, p. 99-114, 2006.

JAIMES, A.; MARTINEZ, S.; COELLO COELLO, C. AN INTRODUCTION TO MULTIOBJECTIVE OPTIMIZATION TECHNIQUES. In: GASPAR-CUNHA, A.; COVAS, J. ed. *Optimization in Polymer Processing*. New York, Nova Science Publishers, 2010, cap 1, p. 1-29.

KAIO, N.; OSAKI, S. Optimal inspection policies: A review and comparison. *Journal of Mathematical Analysis and Applications*, v. 119, n. 1-2, p. 3-20, 1986.

KANG, H. G.; KIM, H. E. Unavailability and spurious operation probability of k-out-of-n reactor protection systems in consideration of CCF. *Annals of Nuclear Energy*, v. 49, p. 102-108, 2012.

KANG, J.; GUEDES SOARES, C. An opportunistic maintenance policy for offshore wind farms. *Ocean Engineering*, v. 216, p. 108075, 2020.

KELLER, A. *Multi-Objective Optimization in Theory and Practice I: Classical Methods*. Sharjah: Bentham eBooks, 2017.

KHATAB, A.; AGHEZZAF, E.; DIALLO, C.; DJELLOUL, I. Selective maintenance optimization for series–parallel systems alternating missions and scheduled breaks with stochastic durations. *International Journal of Production Research*, v. 55, n. 10, p. 3008-3024, 2017a.

KHATAB, A.; AGHEZZAF, E.; DJELLOUL, I.; SARI, Z. Selective maintenance optimization for systems operating missions and scheduled breaks with stochastic durations. *Journal of Manufacturing Systems*, v. 43, Part 1, p. 168-177, 2017b.

KHATAB, A.; AGHEZZAF, E.-H. Selective maintenance optimization when quality of imperfect maintenance actions are stochastic. *Reliability Engineering and System Safety*, v. 150, p. 182-189, 2016.

KHATAB, A.; DIALLO, C.; AGHEZZAF, E.; VENKATADRI, U. Joint optimization of the selective maintenance and repairperson assignment problem when using new and remanufactured spare parts. *IFAC-PapersOnLine*, v. 52, n. 13, p. 1063-1068, 2019.

KHATAB, A.; DIALLO, C.; AGHEZZAF, E.-H.; VENKATADRI, U. Optimization of the integrated fleet-level imperfect selective maintenance and repairpersons assignment problem. *Journal of Intelligent Manufacturing*, DOI: 10.1007/s10845-020-01672-0, 2020.

KHATAB, A.; DIALLO, C.; VENKATADRI, U.; LIU, Z.; AGHEZZAF, E. Optimization of the joint selective maintenance and repairperson assignment problem under imperfect maintenance. *Computer and Industrial Engineering*, v. 125, p. 413-422, 2018.

LASZCZYK, M.; MYSZKOWSKI, P. Survey of quality measures for multi-objective optimization: Construction of complementary set of multi-objective quality measures. *Swarm and Evolutionary Computation*, v. 48, p. 109-133, 2019.

LIN, D.; ZUO, M.; YAM, R. General sequential imperfect preventive maintenance models. *International Journal of Reliability, Quality and Safety Engineering*, v. 7, n. 3, p. 253-266, 2000.

LIMA, V. H. R.; CAVALCANTE, C. A. V.; KRAMER, R. Problema de Manutenção Seletiva e Designação de Mantenedores com Ações Imperfeitas em Sistemas *k-out-of-n*. LIII Simpósio Brasileiro de Pesquisa Operacional, 2021.

LIU, L.; HUANG, H. Optimal Selective Maintenance Strategy for Multi-State Systems Under Imperfect Maintenance. *IEEE Transactions on Reliability*, v. 59, n. 2, p. 356-367, 2010.

LUST, T.; ROUX, O.; RIANE, F. Exact and heuristic methods for the selective maintenance problem. *European Journal of Operational Research*, v. 197, n. 3, p. 1166-1177, 2009.

MAAROUFI, G.; CHELBI, A.; REZG, N. Optimal selective renewal policy for systems subject to propagated failures with global effect and failure isolation phenomena. *Reliability Engineering and System Safety*, v. 114, p. 61-70, 2013.

MAILLART, L.; CASSADY, C.; RAINWATER, C.; SCHNEIDER, K. Selective Maintenance Decision-Making Over Extended Planning Horizons. *IEEE Transactions on Reliability*, v. 58, n. 3, p. 462-469, 2009.

MALIK, M. Reliable Preventive Maintenance Scheduling. *AIIE Transactions*, v. 11, n. 3, p. 221-228, 1979.

MIETTINEN, K. *NONLINEAR MULTIOBJECTIVE OPTIMIZATION*. New York: Springer Science+Business Media, 1999.

MLADENOVIĆ, N.; HANSEN, P. Variable Neighborhood Search. *Computers & Operations Research*, v. 24, n.1, p. 1097-1100, 1997.

PANDEY, M.; ZUO, M.; MOGHADDASS, R. Selective maintenance modeling for a multistate system with multistate components under imperfect maintenance. *IISE Transactions*, v. 45, n. 11, p. 1221-1234, 2013b.

PANDEY, M.; ZUO, M.; MOGHADDASS, R. Selective maintenance scheduling over a finite planning horizon. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, v. 230, n. 2, p. 162-177, 2016.

PANDEY, M.; ZUO, M.; MOGHADDASS, R.; TIWARI, M. Selective maintenance for binary systems under imperfect repair. *Reliability Engineering and System Safety*, v. 113, p. 42-51, 2013a.

PINJALA, S.; PINTELON, L.; VEREECKE, A. An empirical investigation on the relationship between business and maintenance strategies. *International Journal of Production Economics*, v. 104, n. 1, p. 214-229, 2006.

QUDDOOS, A.; ALI, I.; MASOOD KHALID, M. Bi-Objective Fuzzy Selective Maintenance Allocation Problem. *American Journal of Mathematical and Management Sciences*, v. 34, n. 4, p. 289-308, 2015.

RAJAGOPALAN, R.; CASSADY, C. An improved selective maintenance solution approach. *Journal of Quality in Maintenance Engineering*, v. 12, n. 2, p. 172-185, 2006.

RICE, W.; CASSADY, C.; NACHLAS, J. Optimal Maintenance Plans under Limited Maintenance Time. In: Proceedings of the 7th Industrial Engineering Research Conference, Banff, 1998.

RICE, W.*Optimal selective maintenance decisions for series systems*. Starkville, 1999. PhD thesis. Mississippi State University, Department of Industrial Engineering.

RIFAI, A.; NGUYEN, H-T.; DAWAL, S. Multi-objective adaptive large neighborhood search for distributed reentrant permutation flow shop scheduling. *Applied Soft Computing*, v. 40, p. 42-57, 2016.

RIPON, K.; GLETTE, K.; KHAN, K.; HOVIN, M.; TORRESEN, J. Adaptive variable neighborhood search for solving multi-objective facility layout problems with unequal area facilities. *Swarm and Evolutionary Computation*, v. 8, p. 1-12, 2013.

SCHNEIDER, K.; CASSADY, C. Evaluation and comparison of alternative fleet-level selective maintenance models. *Reliability Engineering and System Safety*, v. 134, p. 178-187, 2015.

SCHNEIDER, K.; CASSADY, C. Fleet performance under selective maintenance. In: Annual Symposium Reliability and Maintainability, Los Angeles , p. 571-576, 2004.

WAEYENBERGH, G.; PINTELON, L. A framework for maintenance concept development. *International Journal of Production Economics*, v. 77, n. 3, p. 299-313, 2002.

WAEYENBERGH, G.; PINTELON, L. Maintenance concept development: A case study. *International Journal of Production Economics,* v. 89, n. 3, p. 395-405, 2004.

XU, Q.; GUO, L.; WANG, N.; FEI, R. Recent advances in selective maintenance from 1998 to 2014. *Journal of Donghua University (English Edition)*, v. 32, n. 6, p. 986-994, 2015.

YAVUZ, T.; KUNDAKCIOGLU, O.; ÜNLÜYURT, T. Exact and heuristic approaches to detect failures in failed k-out-of-n systems. *Computers and Operation Research*, v. 112, p. 104752, 2019.

ZHANG, L.; ZHANG, L.; SHAN, H.; SHAN, H. Selective maintenance process optimization based on an improved gravitational search algorithm, from the perspective of energy consumption. *Engineering Optimization*, v. 52, n. 8, p. 1401-1420, 2020.

ZHAO, J.; ZENG, J. Maintenance strategy for stochastic selective maintenance of a two-state system. *International Journal of Wireless and Mobile Computing*, v. 11, n. 4, p. 302-308, 2016.

ZHAO, X.; AL-KHALIFA, K.; HAMOUDA, A.; NAKAGAWA, T. Age replacement models: A summary with new perspectives and methods. *Reliability Engineering and System Safety*, v. 161, p. 95-105, 2017.

ZIO, E.; COMPARE, M. Evaluating maintenance policies by quantitative modeling and analysis. *Reliability Engineering and System Safety*, v. 109, p. 53-65, 2013.