UNIVERSIDADE FEDERAL DE PERNAMBUCO
CENTRO DE TECNOLOGIA E GEOCIÊNCIAS
DEPARTAMENTO DE ENGENHARIA DA PRODUÇÃO
PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA DA PRODUÇÃO

JOSÉ IRANILDO BARBOSA SALES DA SILVA

LEARNING IN A HIRING LOGIC AND OPTIMAL CONTRACTS

Recife
2022

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Thesis presented to the Graduate Program in Production Engineering at the Federal University of Pernambuco, as a partial requirement for obtaining the title of Master or Doctor in Production Engineering.

Area of Concentration: Operations Research.

Advisor: Prof. Dr. Francisco de Souza Ramos.

Recife

2022

Catalogação na fonte:
Bibliotecária Sandra Maria Neri Santiago, CRB-4 / 1267

J586l Silva, José Iranildo Barbosa Sales da.

Learning in a hiring logic and optimal contracts / José Iranildo Barbosa Sales da Silva. – 2022.

76 f.: il., figs., tabs.

Orientador: Prof. Dr. Francisco de Souza Ramos.

Tese (Doutorado) – Universidade Federal de Pernambuco. CTG. Programa de Pós-Graduação em Engenharia da Produção. Recife, 2022.

Inclui referências.

1. Engenharia de produção. 2. Aprendizado. 3. Contratação. 4. Teoria dos jogos. 5. Modelo do principal-agente. I. Ramos, Francisco de Souza (Orientador). II. Título.

UFPE

658.5 CDD (22. ed.) BCTG/2022-421

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Aprovada em: 24/10/2022.

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ACKNOWLEDGMENTS

First of all, I thank God, because He was always with me, regardless of the situation, and I could feel His hand guiding me.

I am immensely grateful to Maria Gerliane Rodrigues Fonseca Cavalcante, who never hesitated and was always by my side. My best friend, partner, companion, girlfriend, without her I know I wouldn't make it this far. She did believe it, when no one else did.

I thank my parents for their support in difficult times. And I thank my sister for always being with me and understanding me.

I would like to thank my supervisor, Professor Francisco de Souza Ramos, who knew how to guide me and direct my skills until this moment. With his profile, he knew how to align myself with the program's objectives.

I would like to thank the Graduate Program in Production Engineering, which pushed me to the limit, making me a high-level professional and researcher, concept 7 in QUALIS CAPES.

ABSTRACT

The concept of learning has always been a fascinating factor in scientific analysis and investigation, and game theory has as its basic instrument the interpretation of the various factors that influence the decision-making process of agents involved in the game. Utilities, perceptions, preferences and decisions have been the subject of research and analysis around the world in the last two centuries. In addition, employment contracts are being formed daily for the most different branches of activity with completely different demands and offers in terms of quantity and variability. Markets also interfere in the hiring logic, as they reflect the bargaining power of each individual inserted in this context of strategic interaction. Therefore, this study involves exactly strategic interaction models that structure the intentions and preferences of decision makers in the game. The principal-agent model, classically known for structuring contracts in search of optimality, will be modified by introducing the concept of learning in non-linear, repeated versions and with cycles of economic interference in player preferences, and, accordingly, will be developed and analyzed the non-linear and repeated learning models of the main agent bringing very strong results for the research such as variation of gains and costs of the principal and agents by the insertion of learning as can be observed in the proposed model and guiding new ways to model the employment contracts for players who always learn with the scenario.

Keywords: learning; hiring; game theory; principal agent model.

RESUMO

O conceito de aprendizagem sempre foi um fator fascinante na análise e investigação científica, e a teoria dos jogos tem como instrumento básico a interpretação dos diversos fatores que influenciam o processo decisório dos agentes envolvidos no jogo. Utilidades, percepções, preferências e decisões têm sido objeto de pesquisas e análises em todo o mundo nos últimos dois séculos. Além disso, os contratos de trabalho estão sendo formados diariamente para os mais diversos ramos de atividade com demandas e ofertas completamente diferentes em termos de quantidade e variabilidade. Os mercados também interferem na lógica de contratação, pois refletem o poder de barganha de cada indivíduo inserido nesse contexto de interação estratégica. Portanto, este estudo envolve exatamente modelos de interação estratégica que estruturam as intenções e preferências dos tomadores de decisão no jogo. O modelo principal-agente, classicamente conhecido por estruturar contratos em busca de otimalidade, será modificado introduzindo o conceito de aprendizagem em versões não lineares, repetidas e com ciclos de interferência econômica nas preferências dos jogadores e, nesse sentido, será desenvolvido e analisarão os modelos de aprendizagem não linear e repetido do agente principal trazendo resultados muito fortes para a pesquisa, tais como, variação dos ganhos e custos do principal e dos agentes pela inserção da aprendizagem como poderá ser observador no modelo proposto e orientando novas formas de modelar os contratos de trabalho para jogadores que sempre aprendem com o cenário.

Palavras-chave: aprendizado; contratação; teoria dos jogos; modelo do principal-agente.

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SUMARY

1 INTRODUCTION

The optimal formulation of labor contracts has always been a topic of discussion regarding the incentives present in the relationship between the agents involved in the hiring process, due to the series of factors present in the scenario. Game theory can help us understand the main issues involved in this model to analyze the different scenarios present in this context (Corgnet, *et al.*, 2018).

In recent decades, several researchers Bejarano *et al*., (2019), Batalgi *et al.*, (2017), Vasconcelos *et al*., (2018), Feng *et al.*, (2017) studied the behavior of the relationship between employers (principal) and employees (agents) in a hiring logic and optimal contracts of labor worldwide. As a result, many questions have been raised, such as the incentives found in each relationship, the kind of information presented in the game, the possibility of controlling the task delegation of hiring to different kinds of agents, and the behavior of the preferences. Understanding these dynamic incentives is central to decision-making.

According to Shattal *et al*., (2017) the most game-theoretic research relies on the availability of spectrum statistics in order to formulate the game and cope with spectrum dynamic changes, especially in stochastic Xu *et al.*, (2012) and repeated games Xiao *et al*., (2012). Such information is not known a priori, limiting the applicability of this approach Xu *et al*., (2013).

In hiring logic, an account must be taken of the degree of agents' global perception of the scenario in which they are involved. In other words, what is needed to understand how much the players learn from basic differences in the scenario, such as changes in performance to a greater perception of the game, understanding the gains of other players, the greater the degree of effort needed to perform a certain task, understanding changes in the labor market, etc. This corroborates the fact that there have been changes in the agent's preferences, which modifies how to draw up an optimal contract for each situation.

Therefore, models with dynamic contracts with limited commitment Kocherlakota (1996), Phelan (1995), Thomas & Worrall (1998) are one way to investigate problems of optimal contracting in environments where one or more of the contracting parties are exposed to outside opportunities that interfere with the continuation of relationship.

On the other hand, contract theory, which focuses on non-monetary incentives, is set out by Corgnet *et al.*, (2018), which introduces goal setting into the hiring scenario and demonstrates that monetary incentives are not a unique motivation tool. Moreover, Pouryousefi

& Frooman (2017) criticize the unilateralism of agency theory and seek a way to formulating contracts bilaterally and Daske (2016) considers a principal-multiple agent model in which agents are privately informed about their intrinsic motivations for collaborating or for competing.

Furthermore, hiring processes have a long tradition of explicit and implicit human biases, which may lead to different consequences Vasconcelos *et al*., (2018). There are several kinds of bias Kiviat (2019), Daske (2016) that may influence hiring decisions. In all these scenarios, the relationship between the players is based on the preference structure of the agent involved in the process. Thus, an analysis must be conducted regarding the consequences of different types of changes in the model.

To analyze this context, we use the principal–agent theory, which has been a very influential theoretical apparatus for gaining insights into the design of labor incentive contracts Chapman (2003), Garen (1994), Laffont & Tirole (1993), Sappington (1991), Stroh *et al.*, (1996). In most principal-agent relationships, the principal has to induce the agent to engage in several tasks simultaneously.

The agent's performance can often be measured fairly accurately in some tasks, but in others, the available performance measures may be very noisy, or a verifiable performance measure that can be used to provide explicit incentives to the agent may not even exist.

For example, a production worker may have to produce a certain amount of output that is easily measurable, but he may also have to ensure that the quality of output is high and that the machinery he is working with is properly maintained, which may be more difficult to monitor.

Another example is a schoolteacher who has to teach his students basic skills, such as the three R's (reading, writing, arithmetic), which can be measured in standardized tests, but also has to stimulate their creativity and teach them social skills, which are much harder to evaluate (Ernst & Schmidt, 2004).

Therefore, the starting point of incentive theory in a labor contract corresponds to the problem of delegating a task to an agent with private information. This private information can be of two types: either the agent can take an action unobserved by the principal called hidden action, or the agent has some private information about its cost or valuation that the principal is unaware of, called hidden knowledge (Laffont & Martimort, 2001).

When two parties engage in a business relationship, their interests are usually not perfectly aligned, and information asymmetry can further exacerbate the tension between them. The principal-agent model is a stylish framework for studying such a problem Zhang & Zenios

(2008). Furthermore, the agents performing the task incur marginal costs that are associated with the number of units of work to be carried out (Laffont & Martimort, 2001).

However, according to the theory of labor Jevons (1957), the marginal cost of doing one more one unit of any activity increases by degrees i.e. the more the individual works the greater the marginal cost of performing a certain activity, which Jevons (1957) called the marginal disutility of labor.

Therefore, the agent’s utility is affected by the increase in the total cost of production, and utility decreases because the rate of pleasure increases, which changes the strategies the principal must use to draw up the contract.

Moreover, in a principal-agent situation, the agent chooses an action "on behalf of" the principal. The consequence of this depends on the random state of the environment as well as on the agent's action. After observing the consequence, the principal makes a payment to the agent according to a pre-announced reward function, which depends directly only on the consequence observed (Radner, 1985).

Another approach to increasing efficiency is the theory of repeated games. If a game with two or more players is repeated, the resulting situation can be modeled naturally as a "supergame" in which the players' actions in any one repetition are allowed to depend on the history of the previous repetitions. In the principal-agent situation, the repetition of the game gives the principal an opportunity to observe the results of the agent's actions over a number of periods and use a statistical test to infer whether or not the agent was choosing the appropriate actions.

The repetition of the game would also provide the principal with opportunities to "punish" the agent for apparent departures from the appropriate actions.

The nonlinearity and the repetition included in a labor contract, using the principal- agent model, changes the structure of the utility functions for all the agents involved in the game.
Repetition is possible because when a principal observes the agent’s performance, his observation is not conclusive, and it may vary his performance in the next hiring due to the individual’s fluctuations that arise from the perception of changes in the market and in their behavior. In addition, the nonlinearity is justified because of the theory set out by Jevons (1957) in the marginal disutility of labor. In both cases, the players involved in the hiring logic learn from the changes imposed by the scenario, which can alter the terms of the optimal contract proposed by (Laffont & Martimort, 2001). In this sense, this thesis verifies the various possibilities of changing perceptions and, consequently, preferences, creating new alternatives

for strategic interaction between the agents involved in the game, causing changes in the formulation of optimal contracting, as will be seen in the next chapters.

1.1 THESIS MOTIVATION

The concept of incentives related to the degree of learning has always permeated the decision-making process in economics. According to Laffont & Martimort (2001), the concept of incentives is inseparable from the main activities that make up current economic behavior. It can be mentioned that for almost everything there should be incentives, incentives to study, incentives to invest, incentives to save. In this sense, this topic becomes central and the object of analysis and questioning around the world.

Furthermore, according to Laffont & Martimort (2001), in a competitive market, companies aim to maximize their profits and this implies a cost minimization problem that necessarily involves all incentives directly or indirectly related to the problem in question.

Usually, when relating incentive and learning together with economic models, a lot of information is partially received and in this game of private information there are a large number of strategic interactions that can be glimpsed. There are many economic environments where private information is a crucial feature, and a key issue is how to provide incentives in a dynamic environment. However, the analysis of dynamic models of hidden information quickly becomes complex, even more so when some of the relevant state variables cannot be monitored (Williams, 2015).

In short, delegating a task to an agent that has objectives different from those envisioned by the principal delegating it can become problematic when information about the agent is imperfect. This is the essence of incentive questions. If the agent had a different objective function but no private information, the principal could propose a contract that perfectly controls the agent and induces the agent's actions to be what it would like to do in a world without delegation. Conflicting goals and decentralized information are therefore the two basic ingredients of incentive theory. (Laffont & Martimort, 2001).

So, exemplifying the context, the starting point of the theory of incentive corresponds, therefore, to the problem of delegating a task to an agent with private information. This private information can be of two types: or the agent can perform an action not observed by the principal, in the case of moral hazard or hidden action; or the agent has some private information about its cost or valuation that is ignored by the principal, in the case of adverse selection or hidden knowledge.

According to Williamson (1975) informational problems have always been the major obstacle for society in general to efficiently allocate all necessary resources. Information asymmetry causes additional costs due to the strategic behavior of agents involved in the model, who tend to want to maximize their benefits according to the informational set present at the time of decision making.

Therefore, the study that analyzes variations in perception and, consequently, in the preferences of the agents involved is important, as there is a latent need to understand the set of information present at the time of decision making, that is, adapted to this context in the time of optimal contractual formulation for all agents involved. This scenario features the natural dynamics of an environment in which the information set is not perfect for the agents involved, which makes it necessary to understand the different variations of perception and preference according to the models that will be shown a posteriori. For this reason, Laffont & Martimort (2001) traced a classic and extremely comprehensive model of what optimality would be in the hiring concept.

In this model, two types of information problems will be considered: adverse selection and moral hazard. Each of these informational problems leads to a different paradigm and, possibly, a different type of agency costs. In addition to the usual technological constraints of neoclassical economics, these agency costs incorporate the information constraints faced by the principal at the time of drafting the contract.

Understanding the possible allocative distortions in a contracting scenario in which the activity delegate does not have complete information about the contracting parties, the scenarios and other variables involved becomes a very favorable environment for the analysis of different types of interventions that may be observed. in the later chapters. The interactions will consider different scenarios that will bring significant repercussions to the classic model of agency theory proposed by Laffont & Martimort (2001).

Analyzing all possible natural changes from the simple perception that each agent involved in the hiring game constantly learns from himself, from his actions, from the actions of other players and from the environment in which he is inserted. Interest in investigating how optimal hiring models can be modified and readapted for agents whose basic utilities have the dynamics of learning is perceptible.

It is known that the concept of learning is dynamic and can arise from different sources at different times of the strategic interaction and this must be incorporated in some way into the logic of hiring in order to minimize the impacts of the immobilization of the non-perception of this agent who learns changes his preferences constantly and this needs to be inserted into the model to check for changes in the optimal situation of the employment contract.

These changes can bring new ways of understanding optimal hiring, new ways of formulating strategies to make the contracting party's objective, which is hiring, in order to maximize its own benefits and, with that, bring more current and dynamic models according to the hiring logic. is thought of nowadays. Where dynamic and learning agents tend to change their perceptions and, consequently, their actions at all times according to each perceived and learned change.

1.2 THESIS OBJECTIVES

This section was created to address the main objectives related to the thesis.

1.2.1 General Objectives

The main objective of this study is to evaluate the changes that occurred and resulting from the changes in the perception of the agents involved in a scenario of task delegation and hiring, given the learning they acquired in the scenarios in which the dynamics becomes more present and consistent with the reality of current hiring.

1.2.2 Specific Objectives

- Evaluate the changes in the formulation of optimal hiring and its consequent repercussions in the context of adverse selection and moral hazard, when the concept of marginal disutility of work is inserted in the utility of the agents involved in the game;
- Understand the scenario in which changes in contractual optimality occur when the perceptions of the players involved are altered due to the transformation of the contracting model into a sequential game;

- Discuss about the modifications of the optimal contractual formulation in which the learning concepts are inserted in the game due to the marginal disutility of the work jointly modeled in a sequential and dynamic context;
- Envision how market changes and economic cycles of expansion and recession change the perceptions of the players involved and modify the optimal contractual scenario by inserting learning from macroeconomic aspects.
- To demonstrate the main results found with the modifications of the traditional model, emphasizing the gains and costs of each player in each proposed situation.

1.3 THESIS STRUCTURE

The thesis is structured in 4 chapters. Chapter I (Introduction) makes an initial understanding and a glimpse of the concept of incentive theory and the factors directly related to it, as well as brings examples of real mechanisms that deal with the theory of incentives. In addition, it outlines the motivation and importance of the study, as well as the main objectives related to the thesis.

Chapter II (Literature Review) deals with the classical principal agent theory proposed by Laffont & Martimort, serving as an initial foundation for understanding the possible variations proposed by this study. In this chapter, the most relevant and important concepts of the principal agent theory will be worked on point by point, making a systematic review of the current literature.

Chapter III (Learning in a Hiring Logic and Optimal Contracts) will deal with the main concepts of learning aimed at contracting logic, where the non-linearity of the utility function of the agents involved and the introduction of exponential costs associated with the act of work. At this point, the concept of Jevons (1957) will be inserted, which deals with the Marginal Disutility of Work and it can be seen how this concept strongly changes the precepts of optimality exposed by the classical theory of incentives. In addition, it will introduce the analyzes of Radner (1985) and will deal with the possibility of understanding the game related to its repetition. At this point, the model ceases to be a “one shot game” and becomes always played, and at this point the concepts of variation of understanding and/or perception by the direction of the game emerge.

With this, it will also bring the non-linearity of the utilities of the agents involved in the game to the context of repeated formulation for infinite horizons and for the informational levels of adverse selection and moral effect, modifying once again the concept of strategic interaction present in the theory of incentives.

In addition, chapter III will provide a critical conceptual analysis of the main learning concepts for economic cycles of employability focused on the context of delegating activities and how these concepts can be incorporated into the main agent models. Finally, chapter IV (Conclusion) will deal with a general overview of the concept of learning aimed at optimal contracts and a conclusion on how the hiring logic is influenced by all the factors highlighted above and inserted in a constantly changing economic context.

2 LITERATURE REVIEW

The Chapter 2 will deal with the main theoretical apparatus necessary for the construction of the model proposed by this thesis.

2.1 LEARNING IN THE THEORY OF INCENTIVES

The concept of learning is directly linked to what incentives are being given to the individual so that he can be stimulated to learn something. Human beings also learn inherently, since learning is a latent characteristic of all animal species.

According to animal learning theory, rewards have three basic functions in behavior. First, they induce learning, as they bring subjects back for more (positive reinforcement); second, they induce consummating approach and behavior to acquire the reward object; and third, they induce positive emotions, which are inherently difficult to investigate in animals. Rewards can serve as behavioral goals if the reward and the contingency between action and reward are represented in the brain during action (Dickinson & Balleine, 1994). In contrast, punishers induce learning avoidance, withdrawal behavior, and negative emotions.

If learning is related to incentives or rewards that the agent can earn, it has stimuli to always be understanding its scenario and what happens in the environment to extract as much information as possible that increases its level of learning and can provide it with an advantage. competitive in strategic interactions with other individuals.

Learning requires temporal contiguity between the conditioned stimulus, or movement, and the reinforcer, as well as the more frequent occurrence of the reinforcer in the presence rather than the absence of the conditioned stimulus (contingency). Analysis of learning conditions reveals that fully anticipated rewards do not contribute to learning (Kamin, 1969).

According to game theory and microeconomics, the value of rewards for behavioral reactions and decisions can be evaluated from the multiplicative product of magnitude and probability of the future reward (expected reward value). Furthermore, the delay to the future reward hyperbolically reduces the reward value (Ho, *et al*., 1999). However, the simple product of magnitude, probability and delay does not always explain how individuals value rewards across all probabilities, and the closely related terms 'utility' (Bernoulli, 1954) and 'perspective' (Kahneman, 1984) allow for a better evaluation of the influence of rewards on decision making.

The basic building blocks of decision making that underlie the learning and assessment process also play important roles in decision making in social contexts. However, interactions between various decision makers in a social group exhibit some features (Fudenberg & Levine, 1998).

When a group of players play the same game over and over again, some players may try to train other players. For example, the recipient in an ultimatum game might reject some offers, not as a result of inequality aversion, but to increase his long-term return by penalizing a greedy bidder. To better isolate the effect of social preference, therefore, many of the behavioral studies in experimental games do not allow their subjects to interact with the same partners repeatedly. In reality, however, learning plays an important role, as people and animals interact with the same individuals over and over again (Lee, 2008).

In this sense, it can be attributed that learning, an inherent capacity of the human being and other animal species, is greatly driven by the incentives and/or rewards that may be brought about by this process. That is, it cannot be dissociated from the concept of incentives, which, consequently, is not dissociated from the concept of behavioral development, preferences and, finally, from the decision-making process.

2.1.1 Learning in the principal agent theory

The concept of learning is part of the most diverse ranges of science in general and could not fail to be included in microeconomics, more specifically, in the field of game theory that deals exactly with the intricacies of strategic interaction between individuals. In the case in question, the principal-agent model, object of this study, is also immersed in the concepts of learning and how this interference can influence the results of optimal hiring.

Ideal contracts in principal-agent models often take complicated forms, for example due to the trade-off between offering insurance and incentives. Depending on the exact setup, the optimal contract crucially depends on the preferences of the principal and agent, the properties of the production technology, and the stochastic properties of the revenue process. As an example, take a standard problem of optimal hiring under moral hazard (Hart & Holmstron, 1987).

The fundamental concern of all principal-agent analysis is control. The literature on principal-agent models was developed in economics, where a particular concern was how to create incentives for agents who did not want to take the risk that would be required in a contract that would successfully align the interests of a principal and agent. In political science, a central concern concerns the administrative procedures of Dunlop and James: Modeling and Learning of the Principal Agent, developed by directors to increase the political control of bureaucratic agents and to predispose these agents to existing political preferences (Dumlop & James, 2009).

Numerous empirical studies in widely diverse areas of research suggest that individuals and companies use in practice and individual learning methods similar to those we analyzed. For example, in the industrial organization, Thornton and Thompson (2001) use a dataset on shipbuilding during World War II to analyze learning inside and outside shipyards. They find that learning spillovers are significant and may have contributed more to increased productivity than conventional learning-by-doing effects. Cunningham (2004) uses data from semiconductor factories and finds that companies that are installing significantly new technologies appear to be influenced by social learning. Singh et al. (2010) found similar effects in the open source software industry (Karaivanov, 2010).

In the development literature, Foster & Rosenzweig (1996) use domestic panel data from India on adoption and profitability of high-yielding crop varieties to test the implications of learning by doing and learning from others. They find evidence that the experience of the families themselves and their neighbors increases profitability. Conley and Udry (2010) investigate the role of social learning in the diffusion of a new agricultural technology in Ghana (Karaivanov, 2010).

The principal agent learning model expands on previous strategic learning models, merging the principal's utility considerations into the induction process, when performed in the presence of strategic agents (Boylu, 2009).

To produce a sensible account of learning principal-agent relationships and how decision makers can learn to stack the deck, the assumption of traditional agency theory that information asymmetries and goal conflicts are corrected must be relaxed (Dumlop & James, 2009).

2.2 CONCEPTS OF THE MODEL OF AGENCY

This topic provides a practical-conceptual overview of agency theory and the principal-agent model and their applications around the world.

One of the first works in the literature on agency theory is due to Jensen and Meckling (1976), who focused on owner-run firms. Other relevant contributions were made, among others, by Fama (1980), Demsetz (1983) and Demsetz and Lehn (1985). The most significant problems that can arise from agency relationships are moral hazard and adverse selection. Both problems are caused by information asymmetry, i.e. when at least one party to a transaction has more or better information than the others. Moral hazard occurs when the ex post behavior of the agent is not appropriate, that is, the agent with more information about its actions has an incentive not to behave in accordance with the interests of the principal. On the contrary, in adverse selection models, ex ante information exchange is not adequate, that is, the principal is not informed about a certain characteristic of the agent (Ciliberti, 2010).

Agency theory assumes that individuals are self-interested creatures and addresses the problem of opportunism (Fama, 1980). To avoid opportunism, it is necessary to give the agent incentives to act in the interests of the principal. This can be done by monitoring behavior or rewarding results (Eisenhardt, 1989a). Which of the two alternatives should be chosen depends on its effectiveness and related costs (Ciliberti, 2010).

Bannock *et al.,* (1992) provide a broad definition of the Principal-Agent problem as: The Principal-Agent problem arises in many spheres of economic activity when a person, the principal, hires an agent to perform tasks on his behalf, but cannot guarantee that the agent executes them exactly the way the director would like. The agent's efforts are impossible or expensive to monitor, and the agent's incentives differ from those of the principal.

Note that there are several components to this definition: 1) the agent's actions may incur some cost or disutility to the principal, 2) the principal's and agent's incentives are misaligned, and 3) it is difficult or impossible for the principal to monitor (or restrict) the agent's actions. The inability of the principal to monitor the agent's actions (or perhaps to assess the costs of these actions) is usually linked to the presence of asymmetric information between the principal and the agent (Scott, 2006).

Most of these studies take the main agent problem as the starting point of the analysis, in which the principal is the owner/shareholder of the firm and the agent is the manager/employee of the firm (Alexander, 2006).

Principal-agent relationships are generally formulated as constrained optimization problems. Constraints are of two types: compatibility of incentives implies that the agent chooses a level of effort that maximizes its utility, and participation (also called individual rationality) requires that the agent voluntarily agree to enter the relationship. The first type of constraint involves checking a large, often infinite, number of inequalities. Under some conditions, however, the incentive compatibility constraints can be relaxed into a more tractable form, requiring the agent to select a level of effort where its utility is stationary. This way of solving the main agent's problems is called the first-order approach. It was justified by Mirrlees (1979), Rogerson (1985) and Jewitt (1988), for one-dimensional principal agent models (Desgagné, 1994).

An important issue in analyzing principal-agent relationships is assessing the impact on the ideal contract of the principal's access to additional information. Therefore, much of the literature since Holmstrom (1979) considers the case where the principal may observe more than one relevant signal. In this context, conditions that guarantee the validity of the first-order approach in general are not known (Desgagné, 1994).

With tools borrowed from insurance economics, lead agency theory has allowed political scientists new insights into the role of information asymmetry and incentives in political relationships. This gave us a way of formally thinking about power as modifying incentives to induce actions in the interest of the principal. Lead agency theory has evolved significantly as political scientists have sought to make it more applicable to peculiarly political institutions. In congressional oversight of bureaucracy, increasing emphasis has been placed on negotiating administrative procedures, rather than imposing incentives based on in results, as originally conceived. Awareness of the problem of reliable commitment has driven more dramatic reformulations, in which agents perform their role only when their interests conflict with those of the principal and they are guaranteed some degree of autonomy. (Miler, 2005).

With Williams (2015), in the model, the principal has a production technology and hires an agent whose effort increases production. The product of production increases the principal's assets, from which he withdraws funds for the agent's payment and his own consumption.

Furthermore, the principal-agent paradigm, where a principal has a major stake in the performance of some system but delegates operational control of that system to an agent, has many natural applications in operations management (OM). However, existing principal-agent models are of limited use to OM researchers because they cannot represent the rich dynamic structure required by OM models (Plambeck & Zenios, 2000).

For Yukins (2010), a principal designates an agent to accomplish the principal's goals, presumably because the agent enjoys some comparative advantage in achieving the goals. Inevitably, however, the interests of the agent diverge from those of the principal; if the agent's goals diverge enough, the agent can be said to have a conflict of interest.

Notably, the risk that such a conflict or such divergence of objectives will be material or harm the principal, increases when an asymmetry of information tends in favor of the agent, that is, in situations where the agent holds much more information than the principal, or when a particularly robust "moral hazard" draws the agent from the edges of the principal (Yukins, 2010).

There is considerable literature on managerial myopia and its causes (Laverty, 1996). Narayanan (1985) reports that if an executive has private information about his decisions, he may make short-sighted decisions that are not in the long-term interest of shareholders (Block, 2012).

The reason is that such a manager wants to improve his reputation in the job market (which primarily values the manager's short-term performance) and improve his wages. Narayanan (1985) also demonstrates that this short-term behavior is negatively related to the manager's experience, the duration of his contract and the risk of the company. Campbell and Marino (1994) extend this idea and show that the problem of Managerial opportunism becomes more severe with mobile managers and flexible labor markets.

If the release agent is able to capture many of the benefits while avoiding most of the costs of "releases that go wrong", the public harm can, in the worst cases, be widespread and large. The potential for moral hazard exists when a release agent (whether a private agent or a public agency) is able to avoid liability for harmful outcomes following a release. Avoiding negative consequences provides little incentive for agents to exercise accountability in their conduct, such as taking only revocable actions. It becomes a useful exercise to employ a principal-agent model to investigate the problem of moral hazard in the provision of revocable actions by a private agent (Pauly, 1968 & Arrow, 1970).

Optimal contracts in principal-agent models often take complicated forms, for example, due to the trade-off between insurance provision and incentives. Depending on the exact configuration, the ideal contract crucially depends on the preferences of the principal and the agent, the properties of the production technology and the stochastic properties of the income process (Karaivanov, 2010).

For Karaivanov (2010), one should describe the ideal contract that arises if both contracting parties are fully rational and know all the ingredients of the problem and contracting environment. Then model and analyze the situation where a director (or directors) with no prior knowledge of the environment has to learn what the ideal contract is.

In most principal-agent relationships, the principal must induce the agent to engage in multiple tasks simultaneously. Agent performance can often be measured quite accurately on some tasks, but on others the available performance measures may be too noisy, or a verifiable performance measure that can be used to provide explicit agent incentives may not even exist (Ernst, 2004).

The reason is that if the principal offers high power incentives for a task that is easy to measure and low power incentives for a task where measurement is difficult, then the agent will focus their efforts on the task that is rewarded and disregard the other task for which only small incentives can be offered (Ernst, 2004).

Furthermore, principal agent theory has been a very influential theoretical apparatus in gaining insights into the design of incentive contracts (Chapman and Ward 2003; Garen 1994; Laffont and Tirole 1993; Sappington 1991; Stroh et al. 1996). When delegating a task to the agent, the principal may not be able to observe the agent's true capability in the pre-contract stage and certainly cannot reliably predict the level of effort the agent will actually expend (working hard or avoiding) in the post-contract stage. -contract (Chang, 2014).

The first barrier to hiring, known as the adverse selection problem, can be overcome by offering a menu of contracts that make the agent happy to negotiate (participation constraint) and willing to reveal their true type (incentive compatibility constraint) (Hart & Holmstrom 1987). In contrast, for Laffont & Martimort (2001), the best way to overcome the last barrier, known as the moral hazard problem, is to hold the agent responsible for the financial consequences of his action. However, to what extent can the risk be transferred to the agent? The principal-agent theory suggests that the optimal risk-sharing relationship is defined at the level where the total transaction surplus can be maximized (Chang, 2014).

Principal agent theory is one of the most influential theoretical foundations for contract design. Due to specialization economies, delegation has become an indispensable part of modern economic life. Delegation will inevitably create the agency problem because the agent's actions cannot be verified by third parties and therefore cannot be contracted. The best thing the principal can do is let the agent share a portion of the reward of the agent's efforts, so that the agent is willing to expend its best efforts to increase the surplus. As lucidly explained in Roberts (2007), the nature of risk-sharing in the principal-agent theory is about a better utilization of the principal's risk-absorbing capacity, assuming that the principal has a greater appetite to take risks.

Considering that this line of thinking has generated a voluminous body of literature with focuses that expand to diverse issues such as repeated relationship, multi-agent, multi-principal, reputation effect, and career concerns (Hart 1995), the modeling technique employed remains the same. paradigm.

Still on the Principal-Agent context, consider two individuals who operate in an uncertain environment and for whom risk sharing is desirable. Suppose an of these individuals (known as the agent) must perform an action that the other individual (known as the principal) cannot observe. Suppose this action affects the total amount of consumption or money that is available to be divided between the two individuals. In general, which action is optimal for the agent will depend on the extent of risk sharing between the principal and the agent. The question is, what is the optimal degree of risk sharing given this dependence? (Hart, 1983).

Particular applications of the principal agent problem have been made in the case of an insurer who cannot observe the insured's level of care, in the case of a landlord who cannot observe a tenant farmer's entry decision (partnership); and the case of a business owner who cannot observe the effort level of a manager or worker. (Hart, 1983).

2.3 THE MODEL OF AGENCY

This section was created to address the main topics related to the agency model, widely disseminated in the academic literature.

2.3.1 The Linear Model of Preferences with Unobservable Costs

According to Laffont and Martimort (2001), the underpinnings of the principal-agent model may be described below:

Consider a consumer or a firm (the principal) who wants to delegate to an agent the production of q units of a good. The value for the principal of these q units is S (q) where $S' > 0$, $S'' < 0$ and $S(0) = 0$. The marginal value of the good is thus positive and strictly decreasing with the number of units bought by the principal.

The production cost of the agent is unobservable to the principal, but it is common knowledge that the fixed cost is F and that the marginal cost θ belongs to the set $\theta = (\underline{\theta}, \bar{\theta})$. The agent can be either efficient ($\underline{\theta}$) or inefficient ($\bar{\theta}$) with respective probabilities v and 1-v. In other words, he has the cost function:

$$C(q;\underline{\theta}) = \underline{\theta}q + F \text{ with probability } v \quad (1)$$

$$C(q;\bar{\theta}) = \bar{\theta}q + F \text{ with probability } 1- v \quad (2)$$

We denote by $\Delta\theta = \bar{\theta} - \underline{\theta} > 0$ the spread of uncertainty on the agent's marginal cost. Suppose first that there is no asymmetry of information between the principal and the agent. The efficient production levels are obtained by equating the principal's marginal value and the agent's marginal cost. Hence, first-best outputs are given by the following first-order conditions:

$$S'\left(\underline{q}\right) = \underline{\theta} \quad (3)$$

$$S'(\bar{q}) = \bar{\theta} \quad (4)$$

For a successful delegation of the task, the principal must offer to the agent a utility level which is at least as high as the utility level that the latter obtains outside the relationship (for each value of the cost parameter). We refer to these constraints as the agent's participation constraints.

$$\underline{t} - \underline{\theta}\underline{q} \geq 0 \quad (5)$$

$$\bar{t} - \bar{\theta}\bar{q} \geq 0 \quad (6)$$

Suppose now that the marginal cost θ is the agent's private information and let us consider the case where the principal offers the menu of contracts $\left\{\left(\underline{t}^*, \underline{q}^*\right); \left(\bar{t}^*, \bar{q}^*\right)\right\}$ hoping that an agent with type $\underline{\theta}$ will select $\left(\underline{t}^*, \underline{q}^*\right)$ and an agent with type $\underline{\theta}$ will select instead $\left(\bar{t}^*, \bar{q}^*\right)$. The problem in this situation appears when the efficient type mimics by the inefficient type due to the asymmetric information. Because of this, the principal needs to impose more two constraints to avoid this practice. They called incentive compatibility constraints:

$$\underline{t} - \underline{\theta}\underline{q} \geq \bar{t} - \underline{\theta}\bar{q} \quad (7)$$

$$\bar{t} - \bar{\theta}\bar{q} \geq \underline{t} - \bar{\theta}\underline{q} \quad (8)$$

To understand the structure of the optimal contract it is useful to introduce the concept of information rent. Under complete information, the principal is able to maintain all types of agents at their zero status quo utility level, but this is not being possible anymore in general under incomplete information, at least when the principal wants both types of agents to be active.

Indeed, take any menu $\{(\underline{t}^*,\underline{q}^*);(\bar{t}^*,\bar{q}^*)\}$ of incentive feasible contracts and consider the utility level that a $\underline{\theta}$-agent would get by mimicking a $\bar{\theta}$-agent. By doing so, he would get:

$$\bar{t} - \underline{\theta}\bar{q} = \bar{t} - \bar{\theta}\bar{q} + \Delta\theta\bar{q} \tag{9}$$

This $\Delta\theta\bar{q}$ is the benefit to the efficient agent by the asymmetric information and the levels of utility by the agents may be expressed by:

$$\underline{U} = \underline{t} - \underline{\theta}\underline{q} \tag{10}$$

$$\bar{U} = \bar{t} - \bar{\theta}\bar{q} \tag{11}$$

According to our timing of the contractual game, the principal must offer a menu of contracts before knowing which type of agent he is facing. Therefore, he will compute the benefit of any menu of contracts $\{(\underline{t}^*,\underline{q}^*);(\bar{t}^*,\bar{q}^*)\}$ in expected terms. The principal's problem writes thus as:

$$\text{Max } v\left(S\left(\underline{q}\right) - \underline{t}\right) + (1-v)(S(\bar{q}) - \bar{t}) \tag{12}$$

Using the definition of the information rents $\underline{U} = \underline{t} - \underline{\theta}\underline{q}$ and $\bar{U} = \bar{t} - \bar{\theta}\bar{q}$, we can replace transfers in the principal's objective function as functions of information rents and outputs so that the new optimization variables are now $\{(\underline{U}^*,\underline{q}^*);(\bar{U}^*,\bar{q}^*)\}$.

$$\text{Max } v\left(S\left(\underline{q}\right) - \underline{\theta}\underline{q}\right) + (1-v)\left(S(\bar{q}) - \bar{\theta}\bar{q}\right) - \left(v(\underline{U}) - (1-v)(\bar{U})\right) \tag{13}$$

The principal wishes to solve problem (P), but first is necessary to find what constraints are relevant for the problem. The ability of the efficient agent to mimic the inefficient becomes his constraints of participation always strictly satisfied. Because of this, the constraints of participation of the inefficient agent is relevant in the problem. Furthermore, the incentive compatibility constraint of the efficient agent is also relevant, because only this agent can mimic his real performance. The maximization with respect the both types of agent results in the data below:

Under asymmetric information, the optimal menu of contracts en-tails:

$$S'\left(\underline{q}\right) = \underline{\theta} \quad (14)$$

$$S'\left(\overline{q}\right) = \overline{\theta} + \frac{v}{1-v}\Delta\theta \quad (15)$$

The information rent obtained by the efficient type depends of the production level $\overline{q}$, that is, the efficient type only can mimic until the $\overline{q}$ units. Thus, the principal offers a menu of contract that minimizes the informational rent of the $\underline{\theta}$- agent.

2.3.2 The Linear Model of Preferences with Unobservable Effort

According to the classical principal agent model, we consider an agent who can exert a costly effort e and e can take two possible values that we normalize respectively as a zero effort level and a positive effort of one: [0,1]. Exerting effort e implies a disutility for the agent which is equal to $\psi(e)$ with the normalizations $\psi(0) = 0$ and $\psi(1) = \psi$.

The agent receives a transfer t from the principal and we assume that his utility function is separable between money and effort, $U = u(t) - \psi(e)$, with u(.) being increasing and concave ($u'(.) > 0$ $u''(.) < 0$.

Production is stochastic and effort affects the production level as follow. The stochastic production level $\tilde{q}$ can take two values $\{\underline{q}, \overline{q}\}$, with $\overline{q} - \underline{q} > \Delta q > 0$ and the stochastic influence of effort on production is characterized by the probabilities $Pr(\tilde{q} = \overline{q}/e = 0) = \pi_0$ and $Pr(\tilde{q} = \overline{q}/e = 1) = \pi_1$ with $\pi_1 > \pi_0$. We denote by $\Delta\pi = \pi_1 - \pi_0$, the difference between these two probabilities.

Therefore, by the mere fact of delegation, the principal loses any ability to control those actions when those actions are no longer observable, either by the principal who others the contract.

We now describe incentive feasible contracts in a moral hazard environment. In such an environment, the agent's action is not directly observable by the principal. The principal can only offer a contract based on the observable and verifiable production level, i.e., a function $\{t(\tilde{q})\}$ linking the agent's compensation to the random output $\tilde{q}$. With two possible outcomes $\overline{q}$ and $\underline{q}$, the contract can equivalently be defined by a pair of transfers $\overline{t}$ and $\underline{t}$. t (resp. $\underline{t}$) is the payment received by the agent if the production $\overline{q}$ (resp. $\underline{q}$) realizes.

The neutral risk agent principal's expect utility writes now:

$$V_1=\pi_1(S(\bar{q}) - \bar{t})+(1- \pi_1)(S(\underline{q}) - \underline{t}) \quad (16)$$

If the agent makes a positive effort (e = 1), and

$$V_0=\pi_0(S(\bar{q}) - \bar{t})+(1- \pi_0)(S(\underline{q}) - \underline{t}) \quad (17)$$

If the agent makes no effort (e = 0).

The problem of the principal is now to decide whether to induce the agent to exert effort or not, and if he chooses to do so, which incentive contract should be used.

To each level of effort that the principal wishes to induce corresponds a set of contracts ensuring participation and incentive compatibility. In the classical model with two possible levels of effort, we will say that a contract is *incentive feasible* if it induces a positive ensures the agent's participation.

The corresponding *moral hazard incentive constraint* writes thus as:

$$\pi_1 u(\bar{t})+(1- \pi_1)u(\underline{t})- \psi \geq \pi_0 u(\bar{t})+(1- \pi_0)u(\underline{t}) \quad (18)$$

is the incentive constraint which imposes that the agent prefers to exert a positive effort. However, when he does not exert effort, the agent incurs no disutility of effort and saves an amount ψ.

Still normalizing at zero the agent's reservation utility, the agent's participation constraint writes now as:

$$\pi_1 u(\bar{t}) + (1- \pi_1)u(\underline{t})- \psi \geq 0 \quad (19)$$

According to our timing of the contractual game, the principal must offer a menu of contracts before knowing the level of effort by the agent. Therefore, the expected utility by the principal when he tries to induce effort by the agent is:

$$\text{Max}\,\pi_1\left(\overline{S}-\overline{t}\right)+(1-\pi_1)(\underline{S}-\underline{t}) \quad (20)$$

Since the participation constraint is binding, we also obtain the value of this transfer which is just enough to cover the disutility of effort, namely $t^* = h(\psi)$.

$$V_1=\pi_1\overline{S}+(1-\pi_1)\underline{S}-h(\psi) \quad (21)$$

Had the principal decided to let the agent exert no effort, e = 0, he would make a zero payment to the agent whatever the realization of output. Thereby, the principal would obtain instead a payoff:

$$V_0=\pi_0\overline{S}+(1-\pi_0)\underline{S} \quad (22)$$

Inducing effort is thus optimal from the principal's point of view when $V1 \geq V0$, i.e.: $\pi_1\overline{S}+(1-\pi_1)\underline{S}-h(\psi) \geq \pi_0\overline{S}+(1-\pi_0)\underline{S}$, or to put it differently when:

$$\Delta\pi\Delta S \geq h(\psi) \quad (23)$$

Where $\Delta S=\overline{S}-\underline{S}$.

The left hand side of $\Delta\pi\Delta S \geq h(\psi)$ captures the gain of increasing effort from e = 0 to e = 1. This gain comes from the fact that the return $\overline{S}$, which is greater than $\underline{S}$, arises more often when a positive effort is exerted. The right-hand side of $\Delta\pi\Delta S \geq h(\psi)$ is instead the first-best cost of inducing the agent's acceptance when he exerts a positive effort.

The principal who wants to induce effort must thus choose the contract, which solves the following problem, if the agent is risk neutral:

$$\text{Max}\,\pi_1\left(\overline{S}-\overline{t}\right)+(1-\pi_1)(\underline{S}-\underline{t}) \quad (24)$$

$$\pi_1 u\left(\overline{t}\right)+(1-\pi_1)u\left(\underline{t}\right)-\psi \geq \pi_0 u\left(\overline{t}\right)+(1-\pi_0)u(\underline{t}) \quad (25)$$

$$\pi_1 u\left(\overline{t}\right)+(1-\pi_1)u\left(\underline{t}\right)-\psi \geq 0 \quad (26)$$

2.4 ANALYSIS ABOUT THE CLASSIC PRINCIPAL AGENT MODEL

Understanding and unraveling the main aspects of the agency concept and theory serves as a foundation for any adaptations that will be developed in the course of this study. The clear understanding and learning of the players' behavior and their behavioral stimuli and, consequently, strategic actions provides the necessary framework to envision other horizons more directly related, in view of this research, the daily and dynamic reality of the current hiring logic in the work.

The agency's model then becomes conducive to changes in its structure to permeate previously unquestioned analyzes, which makes this research unique and extremely important for the academic community and for new formulations of microeconomic concepts that permeate practically all work activities. current.

Thus, the information asymmetry cases demonstrated in this chapter serve as a foundation for understanding the standard scenario and kick-start the search for new possibilities for strategic interaction that will be dealt with a posteriori.

3 LEARNING IN A HIRING LOGIC AND OPTIMAL CONTRACTS

The logic of hiring and the theory of agency are microeconomic models that are widespread and cited by academia around the world, at least in the last six decades. These models are the basis for formulating hiring strategies in order to maximize the employer's utility, also bringing gains to individuals who accept the hiring process.

From this model, structured by Laffont & Martimorti (2001), Radner (1985), among others, it can be seen that hiring can be schematized and led to an optimal result. In this context, several authors brought changes and adaptations that corroborated the model and inserted new structures more consistent with each intended reality, as can be seen in the literature review of this thesis.

With this, the concept of learning can also be incorporated and can be seen in Sales & Ramos (2021), for several scenarios such as: non-linearity of preferences and utilities due to the marginal disutility of work, infinite repetition of hiring logic, bringing these factors such as microeconomic changes and changes in the employability cycle of each market reflecting scenarios of economic expansion and recession and how this learning modifies optimal hiring, the latter being investigated in a macroeconomic context.

Therefore, the contributions brought by Sales & Ramos (2021) will be addressed in chapter III of this thesis in order to demonstrate how the hiring logic can have different optimal results from the change by the introduction of learning in hiring scenarios.

3.1 THE NONLINEAR MODEL OF PREFERENCES

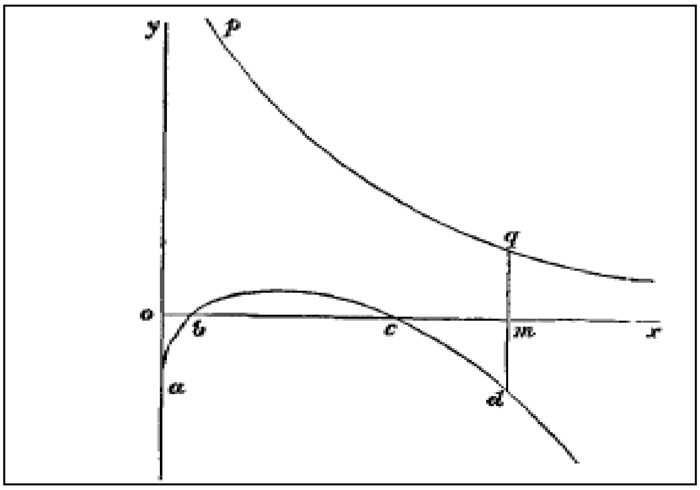
According to Jevons (1957), labor is the painful exertion, which we undergo to ward off pains of greater amount, or to procure pleasures, which leaves a balance in our favor. Labor can also be agreeable at the time and conducive to future good; however, it is only agreeable in a limited amount, and most people are compelled by their wants to exert themselves longer and more severely than they would otherwise do.

Each job requires a different level of effort from the agents. To define this level of effort, Jevons (1957) shows that the amount of labor will be a quantity of two dimensions (intensity and time). Intensity of labor may have more than one meaning; it may mean the amount of work done, or the painfulness of the effort of doing it. Therefore, the theory of labor involves three parameters: the amount of painful exertion, the amount of produce, and the amount of utility gained.

The individual utility of an agent can be modified by using two parameters: Besides the quantity produced, he can vary his utility by the level of effort employed due to the amount of painful exertion. Experience shows that as labor is prolonged, the effort becomes increasingly painful as a general rule. A few hours of work per day may be considered agreeable rather than otherwise; however, as soon as the overflowing energy of the body is drained off, it becomes irksome to remain at work. As exhaustion approaches, continued effort becomes less and less tolerable.

To explain how utility decreases when effort increases because of the increase in the amount of work, some aspects of these phenomena are described in Fig 1.

Figure 1 - Marginal Disutility of Labor



Search: Jevons (1957)

In Fig 1, the height of the points above the line *ox* denotes pleasure and the points below it denotes displeasure. The moment of commencing labor is usually more irregular than when the mind and body are bent well to the work. Therefore, pain was measured using *the oa.* At b, there was neither pain nor pleasure. Between *b* and *c*, an excess of pleasure is represented as being due to exertion. However, after *c*, energy begins to be rapidly exhausted, and the resulting pain is shown by the downward tendency of line *cd.* At the same time, we may represent the degree of utility of the produce by some such curve as *pq*, the amount of produce being measured along the line *ox.*

Therefore, the utility of the agent decreases with increasing rate, whereas the marginal cost increases with decreasing rate. Thus, there is nonlinearity in labor contracts. The principal-agent model is an application of work contracts, and therefore, this model must be analyzed using the marginal disutility of labor concepts.

3.1.1 Learning in the Nonlinear Model

An understanding of an activity is provided by monitoring activity and practice. Hence, players involved in a work contract learn from the daily routine, that is, with the exercise of the function. Therefore, the optimal contract formulation must consider the effort made by the agents and the consequences of this in utility structures, as shown in the model below presented in the topics 3.1.2 and 3.1.3.

3.1.2 Learning with the Labor: Unobservable Costs

According to Laffont & Martimort (2001), a contract can be formulated between an individual who wishes to delegate a particular activity to another agent, but he is not aware of the actual performance of the agent. The incentives of the agents involved in the game are determined by their utility functions that describe the costs and benefits of the players. This contract is designed considering a linear relationship between the costs of carrying out the activity and the amount of labor to be performed, according the classical agency theory elaborated by Laffont & Martimort (2001) and as a way to simplify the approach.

However, the production costs of doing the work may not have a linear relationship with the time or quantity of work performed, because the agents learn from the activity.

With the theory of the marginal disutility of labor, it is demonstrated that the costs of performing a certain activity increases and that to produce q quantities costs the agent less than $3q$, not only because of the quantity realized, but also because of the cost of the growth in the marginal production of q quantities.

Agents can have varying levels of efficiency, with a known assumed probability distribution. It is said, then, that agents can be of different types. By way of simplification of the model the agents involved in this activity are characterized as efficient agent, those who produced larger quantities because they have lower marginal production costs and inefficient agent, which are those who produced smaller quantities because they have higher marginal costs of production.

Thus, the utility functions of the agents involved change as follows:

$$\underline{U} = \underline{t} - (\underline{\theta})^{\alpha}\underline{q} \tag{27}$$

$$\overline{U} = \overline{t} - (\overline{\theta})^{\alpha}\overline{q} \tag{28}$$

- $\underline{U}$ is a high performance agent's utility;
- $\overline{U}$ is a low performance agent's utility;
- $\underline{q}$ is the quantity produced for de efficient agent;
- $\overline{q}$ is the quantity produced for de inefficient agent;
- $\underline{\theta}$ denotes the marginal cost of the efficient agent;
- $\overline{\theta}$ denotes the marginal cost of the inefficient agent;
- $\underline{t}$ is the amount paid for the normal work of an efficient agent;
- $\overline{t}$ is the amount paid for the normal work of an inefficient agent;
- α is a parameter that demonstrates that the cost grows at increasing rates with the realization of an additional unit of work, because of the learning, with $\alpha > 1$.

Moreover, by analyzing the structure developed Laffont & Martimort (2001), it is not easily to visualize the need for changes in the quantity requested by the contracting agent for sporadic and/or floating demands of products and, in several cases, this situation can be found. To introduce the possibility of additional requests appearing, the contract must be drawn up by considering the utility of the agents, whether additional demands do or do not occur. Thus, the utility of the agents is:

$$\underline{U} = (k)\left(\left(\underline{t} + \underline{t_e}\right) - (\underline{\theta})^{\alpha}\underline{q} + \left(\underline{\gamma}^{\beta}\right)\underline{q_e}\right) + (1-k)\left(\underline{t} - (\underline{\theta})^{\alpha}\underline{q}\right) \tag{29}$$

$$\overline{U} = (k)\left((\overline{t} + \overline{t_e}) - (\overline{\theta})^{\alpha}\overline{q} + (\overline{\gamma}^{\beta})\overline{q_e}\right) + (1-k)\left(\overline{t} - (\overline{\theta})^{\alpha}\overline{q}\right) \tag{30}$$

Where:

- k is the probability of the need for additional demand on the part of the principal, and (1 - k) is the complementary probability.
- θ is the marginal cost for each q unit stipulated in the contract for regular hours.
- γ is the marginal cost for each additional unit stipulated in the contract, in which $\gamma - \theta > 0$.
- $\underline{t}$ is the amount paid for the normal work of an efficient agent;
- $\bar{t}$ is the amount paid for the normal work of an inefficient agent;
- $\underline{t_e}$ is the amount paid for the additional work of an efficient agent;
- $\overline{t_e}$ is the amount paid for the additional work of an inefficient agent;
- $\underline{q_e}$ is the additional amount worked by an efficient agent;
- $\overline{q_e}$ is the additional amount worked by an inefficient agent;
- α is the factor that determines the rate of growth of the marginal cost by the increase and effort of the agent in the units defined in the contract because of learning.
- β is the factor that determines the rate of growth of the marginal cost owing to the increase and effort of the agent in the additional units. Given $\alpha > 1$ and $\beta > 1$.

The agents need a minimal benefit to accept the principal's proposal, considering that any external opportunities are equal to zero, that is, any external alternatives are less attractive (less profitable). Thus, new participation constraints are added to those that already exist. Therefore, knowing that the agent accepts the contract of the normal and additional units.

$$(k)\left(\left(\underline{t} + \underline{t_e}\right) - \left(\underline{\theta}\right)^{\alpha}\underline{q} + \left(\underline{\gamma}\right)^{\beta}\underline{q_e}\right) + (1-k)\left(\underline{t} - \left(\underline{\theta}\right)^{\alpha}\underline{q}\right) \geq 0 \tag{31}$$

$$(k)\left(\left(\bar{t} + \overline{t_e}\right) - \left(\bar{\theta}\right)^{\alpha}\bar{q} + (\bar{\gamma})^{\beta}\overline{q_e}\right) + (1-k)\left(\bar{t} - \left(\bar{\theta}\right)^{\alpha}\bar{q}\right) \geq 0 \tag{32}$$

Hence, the agents need to be discouraged from changing productive behavior; thus, the following incentive compatibility constraints are present.

$$(k)\left(\left(\underline{t}+\underline{t_e}\right)-\left(\underline{\theta}\right)^{\alpha}\underline{q}+\left(\underline{\gamma}\right)^{\beta}\underline{q_e}\right)+(1-k)\left(\underline{t}-\left(\underline{\theta}\right)^{\alpha}\underline{q}\right)\geq$$

$$(k)\left(\left(\overline{t}+\overline{t_e}\right)-\left(\underline{\theta}\right)^{\alpha}\overline{q}+\left(\underline{\gamma}\right)^{\beta}\overline{q_e}\right)+(1-k)\left(\overline{t}-\left(\underline{\theta}\right)^{\alpha}\overline{q}\right) \qquad (33)$$

$$(k)\left(\left(\overline{t}+\overline{t_e}\right)-\left(\overline{\theta}\right)^{\alpha}\overline{q}+(\overline{\gamma})^{\beta}\overline{q_e}\right)+(1-k)\left(\overline{t}-\left(\overline{\theta}\right)^{\alpha}\overline{q}\right)\geq$$

$$(k)\left(\left(\underline{t}+\underline{t_e}\right)-\left(\overline{\theta}\right)^{\alpha}\underline{q}+(\overline{\gamma})^{\beta}\underline{q_e}\right)+(1-k)\left(\underline{t}-\left(\overline{\theta}\right)^{\alpha}\underline{q}\right)$$

(34)

Finally, the principal needs to maximize his utility, called *Up*, by formulating a contract that is the most beneficial to him, that is, knowing that there is asymmetry of information and, consequently, it is known that the utility of the principal is:

$$Up = \text{Max } k\left(v\left(S\left(\underline{q}\right)+S\left(\underline{q_e}\right)-\left(\left(\underline{\theta}\right)^{\alpha}\underline{q}+\left(\underline{\gamma}\right)^{\beta}\underline{q_e}\right)\right)+(1-v)\left(S(\overline{q})+S(\overline{q_e})-\left(\left(\overline{\theta}\right)^{\alpha}\overline{q}+(\overline{\gamma})^{\beta}\overline{q_e}\right)\right)\right)+(1-k)\left(v\left(\left(S\left(\underline{q}\right)-\left(\underline{\theta}\right)^{\alpha}\underline{q}\right)\right)+(1-v)\left(S(\overline{q})-\left(\overline{\theta}\right)^{\alpha}\overline{q}\right)\right)$$

(35)

Where:

- S is the benefit that the principal get when the agents produces q units; where $S'(q)>0$, $S''(q)<0$ and $S(0)=0$.

- v is the probability to find efficient agents.

As ***t*** refers specifically to the benefit that accrues to agents from their costs, one can rewrite the utility of the principal by considering costs and information rent, which occurs when the principal is an environment of incomplete information, that is, he wants to delegate an activity to two possible types of agents, but does not know their real productivity.

$$Up = k\left(v\left(S\left(\underline{q}\right) + S\left(\underline{q_e}\right) - \left(\left(\underline{\theta}\right)^{\alpha}\underline{q} + \left(\underline{\gamma}\right)^{\beta}\underline{q_e}\right)\right) + (1-v)\left(S(\overline{q}) + S(\overline{q_e}) - \left(\left(\overline{\theta}\right)^{\alpha}\overline{q} + (\overline{\gamma}^{\beta})\overline{q_e}\right)\right) + v\left(\Delta\theta^{\alpha}\overline{q} + \Delta\gamma^{\beta}\overline{qe}\right)\right) + (1-k)\left(v\left(S\left(\underline{q}\right) + \left(\underline{\theta}\right)^{\alpha}\underline{q}\right) + (1-v)\left(S(\overline{q}) - \left(\overline{\theta}\right)^{\alpha}\overline{q}\right) + v(\Delta\theta^{\alpha}\overline{q})\right) \quad (36)$$

The utility of the principal this in the face of an expected demand, which it will seek to maximize. In addition, based on the probabilities associated with the unawareness of the agents' behavior represented by v and (1-v), the benefits to the principal represented by S $(\underline{q})$ and S $(\overline{q})$, and by the marginal costs of the principal with the contracting of each profile without the possibility of additional demands $\left(\underline{\theta}\right)^{\alpha}\underline{q}$ and $\left(\overline{\theta}\right)^{\alpha}\overline{q}$ and as the possibility of additional demands $\left(\underline{\theta}\right)^{\alpha}\underline{q} + \left(\underline{\gamma}\right)^{\beta}\underline{q_e}$ and $\left(\overline{\theta}\right)^{\alpha}\overline{q} + (\overline{\gamma})^{\beta}\overline{q_e}$. All of them represent the allocative efficiency of the utility of the principal, also possessing the informational rent with and without additional demands $v\left(\Delta\theta^{\alpha}\overline{q} + \Delta\gamma^{\beta}\overline{q}\right)$ and $v(\Delta\theta^{\alpha}\overline{q})$.

The first-order conditions for each agent must be determined because the principal needs to maximize its utility. Given that Up is a strictly concave function, we have a global maximum.

Proposition 1 If there is no possibility of additional demands (i.e., k = 0), the expected results are

$$S'(\overline{q}) = \alpha\left(\overline{\theta}\overline{q}\right)^{\alpha-1}\theta + \frac{v}{1-v}\alpha\left(\Delta\overline{\theta q}\right)^{\alpha-1}\Delta\theta \quad (37)$$

$$S'\left(\underline{q}\right) = \alpha\left(\underline{\theta}\underline{q}\right)^{\alpha-1}\theta \quad (38)$$

Compared with the classic model of Laffont and Martimort (2001), the maximization result for the principal must consider more factors than only $\underline{\theta}$ and $\overline{\theta}$. Given that $\alpha\left(\underline{\theta}\underline{q}\right)^{\alpha-1}\theta > \underline{\theta}$ and $\alpha\left(\overline{\theta}\overline{q}\right)^{\alpha-1}\theta > \overline{\theta}$ with $\alpha > 1$, the principal needs to pay the agents more.

The understanding of the marginal disutility of labor is a driver of the agents' results, which leads to more principal spending.

Proposition 2 If there is the possibility of additional demands, then k > 0, the expected results are as follows:

$$S'\left(\underline{q}\right) = \alpha\left(\underline{\theta q}\right)^{\alpha-1}\theta \quad (39)$$

$$S'(\overline{q}) = \alpha\left(\overline{\theta q}\right)^{\alpha-1}\theta + \frac{v}{1-v}\alpha\left(\Delta\overline{\theta q}\right)^{\alpha-1}\Delta\theta \quad (40)$$

$$S'\left(\underline{q}_e\right) = \beta\left(\underline{\gamma q}\right)^{\beta-1}\gamma \quad (41)$$

$$S'(\overline{q_e}) = \beta\left(\underline{\gamma q}\right)^{\beta-1}\gamma + \frac{kv}{(1-k)(1-v)}\beta(\Delta\overline{\gamma q})^{\beta-1}\Delta\gamma \quad (42)$$

Compared with the classic model of Laffont and Martimort (2001), the maximization result for the principal brought other variables that leverage the agents' marginal costs both in the normal amount worked and in the additional amount. Knowing that $\beta > \alpha$, this disutility is even greater in the additional quantities, which makes the principal reformulate his proposals for the most efficient agent with larger disbursements.

3.1.3 Learning with labor: Effort Level

According to the classical principal-agent model, we consider an agent who can exert a costly effort, and this effort can take two possible values that we normalize as a zero-effort level and a positive effort of one: [0,1]. Exerting effort *e* implies a disutility for the agent that is equal to $\psi(e)$ with the normalizations $\psi(0) = 0$ and $\psi(1) = \psi$. Thus, the positive value was equal to 1. However, when we consider the disutility of the agent as a function $\psi(e)$, we admit that e is a constant value that varies between 0 and 1. Therefore, $\psi(e) = e$.

Using the concept of learning in the marginal disutility of labor, the effort e varies with the production level, and e increases when the level of production increases. Therefore, the disutility function is $\psi(e) = e^{\alpha}$, where α represents the exponential growth of the production level, with $\alpha > 1$. In this sense, e is not constant.

The agent's utility changes to $U = u(t) - e^{\alpha}$, with u(.) increasing and concave (u'(.) > 0 u''(.) < 0.

The principal´s expected utility of the agent is written as:

$$V_1 = \pi_1\left(S(\overline{q}) - \overline{t}\right) + (1 - \pi_1)(S(\underline{q}) - \underline{t}) \quad (43)$$

If the agent makes a positive effort (*e* = 1), and

$$V_0 = \pi_0\left(S(\overline{q}) - \overline{t}\right) + (1 - \pi_0)(S(\underline{q}) - \underline{t}) \quad (44)$$

If the agent makes a negative effort (*e* = 0).

Where:

- $S(\overline{q})$ is principal benefits of a low performance agent's work;
- $S\left(\underline{q}\right)$ is principal benefits of a high performance agent's work;
- $\underline{t}$ is the amount paid for the normal work of an efficient agent;
- $\overline{t}$ is the amount paid for the normal work of an inefficient agent;
- π_1 is the probability associated to level of effort equal to 1;
- π_0 is the probability associated to level of effort equal to zero.

The principal wishes to induce a positive effort (*e* = 1) to maximize his utility, such that $V1 \geq V0$. However, when we introduce the new agent's utility function $U = u(t) - e^{\alpha}$, the exponential disutility is inserted in the context, thus changing the model. Knowing this, $e^{\alpha} = \psi(e)$ only when α = 1.

Given that the production level *q* increases, the effort increases at an increasing rate with α > 1. Therefore, the disutility of the agent is higher than that of the classical model, and α tends to increase with the growth of disutility.

Therefore, the deadline must be found that makes the principal wish to induce the agent to make a positive effort e = 1. Thus, the new *moral hazard incentive constraint* can be written as:

$$\pi_1 u\left(\overline{t}\right) + (1-\pi_1)u\left(\underline{t}\right) - e^{\alpha} \geq \pi_0 u\left(\overline{t}\right) + (1-\pi_0)u\left(\underline{t}\right) \quad (45)$$

This is the incentive constraint, which implies that the agent prefers to exert a positive effort. However, the utility for the positive effort should be greater than that of the classical model to compensate for this new disutility.

Therefore, the principal continues to induce a positive effort, but there is a limit that decreases the desire to induce a greater effort than that of the classical model.

Proposition 1. If the principal induces a positive effort greater than the limit for the exponential disutility, his costs could be greater than the benefits that discourage the principal from taking the action.

Proposition 2. If the disutility α is very high, the principal cannot demand much effort considering the production level *q*, such as increases in *q* increases *e*, which restricts the principal's ability to require the agent to make an effort.

The agent's participation constraint is now re-written as:

$$\pi_1 u(\bar{t}) + (1 - \pi_1)u(\underline{t}) - e^{\alpha} \geq 0 \tag{46}$$

Therefore, the expected utility of the principal when he tries to induce effort by the agent is:

$$\pi_1(\bar{S} - \bar{t}) + (1 - \pi_1)(\underline{S} - \underline{t}) \tag{47}$$

Since the participation constraint is binding, we also obtain the value of this transfer, which is sufficient to cover the disutility of effort, namely $t^{**} = e^{\alpha}$

$$V_1 = \pi_1\bar{S} + (1 - \pi_1)\underline{S} - e^{\alpha} \tag{48}$$

Had the principal decided to let the agent exert no effort, $e = 0$, he would make a zero payment to the agent regardless of the agent’s output. Therefore, the principal obtains payoff as follows:

$$V_0 = \pi_0\bar{S} + (1 - \pi_0)\underline{S} \tag{49}$$

Inducing effort is thus optimal from the principal's point of view when $V1 \geq V0$, that is, $\pi_1\bar{S} + (1 - \pi_1)\underline{S} - h(\psi) \geq \pi_0\bar{S} + (1 - \pi_0)\underline{S}$, or to put it differently when:

$$\Delta\pi\Delta S \geq h(\psi) \tag{50}$$

Where $\Delta S = \bar{S} - \underline{S}$

However, if α is sufficiently large, $V1 \rightarrow V0$. Therefore, there is a value of *α* for each q production level that discourages the agents who want to grow because the disutility is larger than the benefits.

The principal continues to induce effort and must choose the contract, which solves the following problem.

$$\pi_1 \left(\bar{S} - \bar{t}\right) + (1 - \pi_1)(\underline{S} - \underline{t}) \quad (51)$$

$$\pi_1 u\left(\bar{t}\right) + (1 - \pi_1)u\left(\underline{t}\right) - e^{\alpha} \geq \pi_0 u\left(\bar{t}\right) + (1 - \pi_0)u(\underline{t}) \quad (52)$$

$$\pi_1 u\left(\bar{t}\right) + (1 - \pi_1)u\left(\underline{t}\right) - e^{\alpha} \geq 0 \quad (53)$$

Therefore, the amount *t* for both agents is larger than that of the classical model to compensate for the marginal disutility of labor by the agents involved.

Moreover, for production level *q*, *α* tends to increase, thus forcing the principal to reformulate the optimal contract considering the new disutility.

Changing the terms, we have:

$$Up = \text{Max } \pi_1 \left(\bar{S} - \bar{e}^{\alpha}\right) + (1 - \pi_1)(\underline{S} - \underline{e}^{\alpha}) \quad (54)$$

Extracting the first-order conditions from the principal's utility function, we have to

$$\underline{S}' = \alpha \underline{e}^{\alpha - 1} \quad (55)$$

$$\bar{S}' = \alpha \bar{e}^{\alpha - 1} \quad (56)$$

From the information obtained in the first-order conditions, it is possible to define $\alpha e^{\alpha-1} > e$ regardless of the level of effort of the contracted agent, which means that the principal has to disburse *S*, which is a larger amount than the classic model by Laffont & Martimort (2001).

3.1.4 Numerical example and results

This subsection describes the numerical case that demonstrates the impact of learning with labor in the classical model for unobservable costs and levels of effort.

3.1.4.1 Numerical example for Unobservable Costs

Given that in the classical principal-agent model, θ grows linearly with the quantity produced, the principal draws up a contract of execution of n tasks, given $q = n$ for both agents. Knowing that, the efficient agent will produce more that the inefficient agent, the principal distributes the contract menu: {($ 500, x units); ($ 300, x units)}.
The principal does not observe the agents' marginal cost, but the agents know their costs for the inefficient agent θ = $ 10.00, and for the efficient agent θ = $ 5.00. Therefore, we have the following payoffs as described in the Table 1:

Table 1 - Payoff on the Classical Principal Agent Model

Agent	$\underline{\theta}$ – Agent	$\bar{\theta}$ - Agent
Payoff	$ 500 + $ 100	$ 300

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The efficient agent accepts a contract up to x = 100 units produced, because its marginal cost of θ = $ 5.00, increases linearly with the quantity produced. The inefficient agent accepts a contract up to x = 30 units produced. Since the efficient agent can be considered inefficient, he can have an additional $ 100 cash to keep him from concealing his real performance. However, with the marginal disutility of the work proposed by Jevons (1957), keeping the values of t and θ constant, one should take into account the $\alpha > 1$, which for both agents in this model grows in the same way.

Thus, we have the sensitivity analysis below, with increments of 50% in the value of α every 10 units produced for the efficient agent and three units produced for the inefficient agent in the normal production.

Moreover, increments of 50% must be implemented in the value of α every five units produced for the efficient agent and three units produced for the inefficient agent in the additional production. Table 2 presents the new payoffs**.**

Table 2 - Production level on the Learning with labor to Unobservable Costs

Agent	$\underline{\theta}$ - Agent				$\bar{\theta}$- Agent			
	α = 1	α = 1,5	α = 2	α = 2,5	α = 1	α = 1,5	α = 2	α = 2,5
Payoffs	$ 600	$600	$600	$600	$300	$300	$300	$300
Level of Production	100 units	40.25 units	16 units	6,26 units	30 units	8.53 units	2.40 units	0.66 units

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Given that the marginal cost of the efficient agent increased by 50% after the first 10 units produced, it did not produce 90 units, as in the classical model. The new denominator is $5^{1.5}$ = 11.18; because of this, 450 remaining divided by 11.18, which results in 40.25 units produced, a loss of 55.27%.

For the principal to have the 90 units stipulated in the contract, he will have to pay larger amounts. With α = 2, the efficient agent produces another 16 units for the same value of the initial contract with a loss of 80%. With α = 2.5, the agent will produce another 6.26 unit with a loss of 91.05%.

The same occurs with the inefficient agent, he will only produce another 8.53 units if α = 1.5; 2.4 units if α = 2 and 0.66 units if α = 2.5, with, respectively, percentage losses of: 68.40% if α = 1.5; 90% if α = 2 and 96.85% if α = 2.5.

As there is still the possibility of additional demands, the principal draws up a contract menu for that possibility. However, β, which is the increasing marginal cost of the additional activity, is greater than α. Therefore, the principal will have to pay even more to get both agents to perform the task stipulated in the contract.

The contract menu given by the principal is {($ 50 per additional unit); ($ 30 per additional unit)}. However, marginal costs are higher for both the efficient agent θ = $ 6.00, and the inefficient agent θ = $ 12.00. In addition, the efficient agent has more information rent from the emergence of additional demands, as he can be inefficient in this case as well.

Therefore, the principal disburses $ 150 to inhibit this action. All marginal costs of the model are monetary terms that facilitate the demonstration. Therefore, we have:

Table 3 - Production level on the Learning with labor for additional demands to Unobservable Costs

Agent	$\underline{\theta}$ - Agent				$\bar{\theta}$- Agent			
	β = 1.5	β = 2	β = 2.5	β = 3	β = 1.5	β = 2	β = 2.5	β = 3
	α = 1	α =1.5	α = 2	α =2.5	α = 1	α =1.5	α = 2	α = 2.5
Normal Payoff	$750	$750	$750	$750	$300	$300	$300	$300
Additional Payoff	$50	$50	$50	$50	$30	$30	$30	$30
Normal Production	100 units	40.25 units	16 units	6.26 units	30 units	8.53 units	2.4 units	0.66 units
Additional Production	3.06 units	1.11 units	0.39 units	0.13 units	0.72 units	0.18 units	0.04 units	0.01 units

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Therefore, because the marginal cost of the efficient agent increased by 50% after the first five units produced in the additional time, it will not produce 7,5 units, as in the classical model.

The new denominator is $6^{1.5} = 14.69$, resulting in these 45 remaining units divided by 14.69, resulting in 3,06 units produced and a loss of 59.2%. For the principal to have the 7,5 units stipulated in the contract, he will have to pay out larger amounts. With β = 2, the efficient agent will produce another 1.11 units for the same value of the initial contract with a loss of 72.25%.

With β = 2,5, the agent will produce another 0,39 units with a loss of 88.85%. With β = 3, the efficient agent will produce another 0,13 units for the same value of the initial contract with a loss of 99.56%. The same occurs with an inefficient agent. He will only produce another 0.72 units if β = 1,5; 0.18 units if β = 2, and 0.04 units if β = 2,5 and 0,01 units if β = 3, with, respectively, percentage losses of: 97,60% if β = 1,5; 99.30% if β = 2; 99.83% if β = 2.5 and 99.95% if β = 3.

3.1.4.2 Numerical Example for Effort Level

For the classical model with moral hazard, the agents exert an effort that is not observed by the principal. This effort can be represented by *e*, which is a parameter of disutility embedded in the utility function of the agents involved in an asymmetric information contract. However, the parameter *e* can vary according to the marginal disutility of labor (Jevons, 1957).

Therefore, the principal distributes the same contract menu of adverse selection {($ 500, x units); ($ 300, x units)} when both agents exert positive effort (*e = 1*). The effort produced by the agents is not observed by the principal, but the agents know their real efforts for the inefficient agent e = 5 and for the efficient agent e = 3. Therefore, we have the following payoffs, as listed in Table 4.

Table 4 - Payoff on the Classical Principal Agent Model

Agent	$\underline{e}$ – Agent	$\bar{e}$ - Agent
Payoff	$ 500 + $ 100	$ 300

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The efficient agent accepts a contract up to x = 200 units produced because of the unobserved effort of *e* = 3. The inefficient agent accepts a contract up to x = 60 units produced because of the unobserved effort of *e* = 5. Since the efficient agent can be considered inefficient, he can have an additional $ 100 cash to keep him from concealing his real performance.

However, with the marginal disutility of the work proposed by Jevons (1957), keeping the values of *t* and *e* constant, one should take into account the $\alpha > 1$, which for both agents in this model grows in the same way.

Thus, we have the sensitivity analysis below, with increments of 50% in the value of α every 10 units produced by the efficient agent and three units produced by the inefficient agent in the normal production. Table 5 presents the new payoffs.

Table 5 - Production Level on the Learning with labor per Effort Level

Agent	$\underline{e}$ – Agent				$\bar{e}$ - Agent			
	α = 1	α =1.5	α = 2	α = 2.5	α = 1	α =1.5	α = 2	α = 2.5
Payoffs	$600	$600	$600	$600	$300	$300	$300	$300
Level of Production	200 units	109.82 units	60 units	32.73 units	60 units	25.49 units	10.8 units	0.91 units

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Given that the effort of the efficient agent increased by 50% after the first 10 units produced, he will not produce 190 units as in the classical model, $3^{1.5}$ = 5,19, that is, 570 remaining divided by 5.19, which results in 109.82 units produced, a loss of 45.09%. For the principal to have 190 units stipulated in contract, he will have to pay larger amounts. With α = 2, the efficient agent will produce another 60 units for the same value of the initial contract with a loss of 66.67%.

With α = 2.5, the agent will produce 32.73 more units with a loss of 81.60%. The same occurs with an inefficient agent. He will only produce another 25.49 units if α = 1.5; 10.8 units if α = 2 and 0.91 units if α = 2.5, with, respectively, percentage losses of: 55.28% if α = 1.5; 80% if α = 2 and 98,21% if α = 2.5.

3.2 LEARNING IN THE REPEATED GAME

In labor relations, the information asymmetries related to the performance of the hired individuals diminish as these contracts are put into practice, since the principal will have a greater understanding of the performance of each employee.

Thus, an analysis needs to be made of how the optimal contract proposed by Laffont & Martimort (2001) behaves with the updating of the contractor's beliefs by viewing the agent’s performance.

Until then, time was not a variable to be considered in games and models aimed at hiring, more specifically principal-agent models, that is, games were characterized as static and players only met once. However, there are situations where players can meet more often and enter the sphere of dynamic games. These dynamic games can be classified in several ways, such as: repeated, non-repeating, deterministic, stochastic, finite horizon, infinite horizon, etc.

According to Fudenberg & Tirole (1991), sequential (or dynamic) games are games where the next player is aware of his predecessor's move. This doesn't need to be perfect knowledge about every previous player's action; it needs very little information. These games can be characterized by having perfect (symmetrical) or imperfect (asymmetrical) information and it is in this sphere that this chapter works. In this sense, dynamic (repeated) games with imperfect information, as players do not know all the game information at the time of the move.

Dynamic games can be characterized in terms of their time horizon as finite and infinite. According to Fudenberg & Tirole (1991), finite games are those that end in an established (finite) number of steps. All static games are finite as they end in a single step. Dynamic games, on the other hand, can be finite, ending in a given number of steps, or infinite, and can be extended indefinitely.

In the case of the main agent model for sequential and repeated games, the horizon to be delineated will be infinite p, since games with finite horizons can be solved by Perfect Nash equilibrium in Subgames in the same way as simultaneous games and this approach already has been worked on in previous chapters. Therefore, the main repeated agent model for infinite and sequential horizons can be described below.

3.2.1 Learning with the Understanding by Repetition of the Game for Unobservable Costs

The results obtained by learning in the nonlinear preferences force the principal to disburse more to compensate for the disutility of agents involved in a contract. However, when this game is played infinitely, bearing in mind that all the players like a long-run player with the discount factor, the results may be different when the nonlinear model and the classical model are compared.

In a repetition of the game, the agent’s utilities are changed considering the gains and costs for all possibilities of time. Given that time is represented by t.

During each period, the principal and agent played a one-period game with a new random environment each time. In each period, each player's actions depend on what he has observed up to that point in time (Radner, 1985).

For the principal, this is the history of his own previous actions (i.e., announced reward pairs) and the history of previous successes and failures. For the agent, this is the history of his own and the principal's previous actions, the history of previous successes and failures, and the reward pair that the principal has just announced. Neither player observes random environments, which are assumed to be independent and identically distributed. At the end of each period, after observing the current success or failure, the principal compensates the agent according to the reward pair that he announced at the beginning of the period. A supergame strategy for a player is a sequence of decision rules that determine an action at each period as a function of their information history at that point in time.

The supergame payoff for a player is the normalized sum of the discounted expected payoff (Radner, 1985).

When this game is played by $t = 1$, the results are presented in section 2. However, when $t > 1$, some hypotheses must be formulated to define the model.

When $t > 1$, the hypotheses are:

- The game is played infinitely, where for each period of time, the principal formulates a different contract with the possible agents.
- The contract is updated by the set of information made available by the principal for each period when he makes a decision.
- The principal and the agents are long-run players with discount factors, and both need to maximize their payoffs for all games.
- The discount factor determines the type of each player in this game.
- The history defines the future decisions for all players.

3.2.2 Timeline

- At $t = 1$, the principal does not know about the agents' performance and makes a decision using the probabilities for each type of agent.
- In $t > 1$, the principal increases his knowledge about the agent's behavior and starts to understand how the agent's utility function works. Because of this, he updated the probabilities for both agents using Bayes' rule to decrease the possibility of asymmetric information.

3.2.3 The structure of the Game

The principal needs to delegate an activity to someone and he does not know about the agents' real performance, which is a problem that is found in the classical model Laffont & Martimort (2001).

However, the game was repeated. For example, the principal needs to contract someone to carry out a service in his company and in different periods of time, which is offered to different kinds of agents.

In this situation, the principal may analyze the first agent's performance and make suppositions in accordance with the history regarding the probability distribution of the new agent's performance.

As mentioned above, this game is played repeatedly, and therefore, the agents' utility functions change to incorporate the possibility of the repeated game:

$$(1 - \underline{\delta}) \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\underline{t} - \underline{\theta}^{\alpha}\underline{q}\right)) \tag{57}$$

$$(1 - \bar{\delta}) \sum_{t=1}^{\infty} \bar{\delta}^{t-1} (\bar{t} - \bar{\theta}^{\alpha}\bar{q})) \tag{58}$$

Where:

- $\underline{\delta}$ represents the discount factor of the efficient agent;
- $\bar{\delta}$ represents the discount factor of the inefficient agent;
- $\underline{\delta} > \bar{\delta}$, the discount factor for the efficient agent is greater than the discount factor for the inefficient agent because he is more patient than the inefficient player.

The agents need a minimal benefit to accept the principal's proposal, considering that any external opportunities are equal to zero. Thus, new participation constraints are added to those that already exist. Hence, knowing that the agent accepts the contract of normal units:

$$(1 - \underline{\delta}) \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\underline{t} - \underline{\theta}^{\alpha}\underline{q}\right)) \geq 0 \tag{59}$$

$$(1 - \bar{\delta}) \sum_{t=1}^{\infty} \bar{\delta}^{t-1} \left(\bar{t} - \bar{\theta}^{\alpha}\bar{q}\right)) \geq 0 \tag{60}$$

Thus, the agents need to be discouraged from other than usual behavior; therefore, the following incentive compatibility constraints are present.

$$(1 - \underline{\delta}) \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\underline{t} - \underline{\theta}^{\alpha}\underline{q}\right)) \geq (1 - \underline{\delta}) \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\underline{t} - \overline{\theta}^{\alpha}\underline{q}\right)) \quad (61)$$

$$(1 - \overline{\delta}) \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\overline{t} - \overline{\theta}^{\alpha}\overline{q}\right)) \geq (1 - \overline{\delta}) \sum_{t=1}^{\infty} \overline{\delta}^{t-1} \left(\overline{t} - \underline{\theta}^{\alpha}\overline{q}\right)) \quad (62)$$

This structure is applied to avoid the possibility of agents mimicking a contract today and in the future.

Finally, the principal needs to maximize his utility by formulating a contract that is the most beneficial to him in a dynamic infinity game, that is, knowing that there is asymmetry of information and, consequently, resulting information rent for the active restriction agent, it is known that the utility of the principal is

$$\text{Max Up} = p_{h'} \sum_{t=1}^{\infty} \delta_p^{t-1} (S\left(\underline{q}\right) - \underline{\theta}^{\alpha}\underline{q}) + (1 - p_{h'}) \sum_{t=1}^{\infty} \delta_p^{t-1} \left(S(\overline{q}) - \overline{\theta}^{\alpha}\overline{q}\right) + p_{h''} \sum_{t=1}^{\infty} \underline{\delta}^{t-1} (\underline{t} - \underline{\theta}^{\alpha}\underline{q}) + (1 - p_{h''}) \sum_{t=1}^{\infty} \overline{\delta}^{t-1} \left(\overline{t} - \overline{\theta}^{\alpha}\overline{q}\right) + p_{h''} \sum_{t=1}^{\infty} \overline{\delta}^{t-1} \Delta\theta q \quad (63)$$

Where:

- *ph'* is the conditional probability obtained from the history observed by the principal. Thus, the probabilities are updated period by period in the utility function of the principal.

- *ph''* is the conditional probability obtained from the history observed by the agents. Thus, the probabilities are updated periodically in the utility function of the agents.

  All the types of the agents observe the history in the same way.

- The relationship between the discount factors is: $\delta_p > \underline{\delta} > \overline{\delta}$.

- With $0 < \overline{\delta} < \underline{\delta} < \delta_p < 1$.

Given that the principal needs to maximize utility, the first-order condition must be defined. This is shown below:

$$S'\left(\underline{q}\right)=\frac{2-\delta_p-\underline{\delta}}{1-\underline{\delta}}\theta \quad (64)$$

$$S'(\overline{q})=\overline{\theta}+\frac{1-\delta_p}{1-p}\left(\frac{(1-p)\theta}{1-\delta}+\frac{p\Delta\theta}{1-\delta_p}\right)=\frac{1-\delta_p}{1-\overline{\delta}}\theta+\frac{1-\delta_p}{1-\delta}\frac{p}{1-p}\Delta\theta \quad (65)$$

According to the relationship between discount factors, $\delta_p > \underline{\delta} > \overline{\delta}$. $1 - \delta_p < 1 - \overline{\delta}$ and $\frac{1-\delta_p}{1-\delta} < 1$.Therefore, the player in the second period earns less than the player in the first period because of the learning and the decrease in asymmetric information. As $\frac{2-\delta_p-\delta}{1-\underline{\delta}}\theta > 1$.

Thus, there is an income transfer of the lower performance player to the higher performance player and, consequently, to the principal.

3.2.4 Discussion

Given the above, the histories in each period of time for each player may be represented by:

- *h'* – The history observed by the principal when he makes a decision: $h' \in H$, and H is the set of all the histories presented.
- *h''* - The history observed by the agents when he makes a decision: $h'' \in H$, and H is the set of all the histories presented.

In the one-shot game principal–agent model, the probabilities of each agent's type are built *a priori* according to the environment. However, when this game is played infinitely, it is possible that the principal will try to review his beliefs and build, from the observation, probabilities *a posteriori* using Bayes' rule. In this case, probabilities were built based on observations of past histories.

Thus, h'_1 is the history observed by the principal at $t = 2$, and h'_2 is the history observed by the principal at $t = 3$. It is possible to find ph'_1 and $(1 - ph'_1)$ at $t = 2$.
When this is known, p is the probability of contracting the efficient type for $t = 1$. The Bayes rule may be used to define ph'_1, given p.

3.2.5 Learning with the understanding by repetition of the game to effort level

According to the same hypotheses, for the repeated game for adverse selection, when *t > 1*, that is,

- The game is played infinitely, where for each period of time, the principal formulates a different contract with the possible agents.
- The contract is updated by the set of information made available by the principal for each period when he makes a decision.
- The principal and the agents are long-run players with discount factors, and both need to maximize their payoffs for all games.
- The discount factor determines the type of each player in this game.
- The history defines the future decisions for all players.

The utilities of both agents for the moral hazard repeated game is:

$$(1 - \underline{\delta}) \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\pi_1 u(\overline{t}) + (1 - \pi_1) u(\underline{t}) - e^{\alpha}\right) \quad (66)$$

$$(1 - \overline{\delta}) \sum_{t=1}^{\infty} \overline{\delta}^{t-1} \left(\pi_1 u(\overline{t}) + (1 - \pi_1) u(\underline{t}) - e^{\alpha}\right) \quad (67)$$

Where:

- $\underline{\delta}$ represents the discount factor of the efficient agent;
- $\overline{\delta}$ represents the discount factor of the inefficient agent;
- $\underline{\delta} > \overline{\delta}$, the discount factor for the efficient agent is greater than the discount factor for the inefficient agent because he is more patient than the inefficient player.

The principal needs to maximize his utility by formulating a contract that is the most beneficial to him in a dynamic infinity game, that is, knowing that there is asymmetry of information and, consequently, resulting information rent for the active restriction agent, it is known that the utility of the principal is

$$\text{Max Up} = p_{h'} \sum_{t=1}^{\infty} \delta_p^{t-1} \left(\left(\pi_1 u(\underline{t}) - e^{\alpha} + (1-\pi_1) u(\bar{t}) - e^{\alpha} \right) + (1 - p_{h'}) \sum_{t=1}^{\infty} \delta_p^{t-1} \left(\left(\pi_1 u(\underline{t}) - e^{\alpha} + (1 - \pi_1) u(\bar{t}) - e^{\alpha} + p_{h''} \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\left(\pi_1 u(\underline{t}) - e^{\alpha} + (1 - \pi_1) u(\bar{t}) - e^{\alpha} \right) + (1 - p_{h''}) \sum_{t=1}^{\infty} \underline{\delta}^{t-1} \left(\left(\pi_1 u(\underline{t}) - e^{\alpha} + (1 - \pi_1) u(\bar{t}) - e^{\alpha} \right) \right) \right. \right. \right. \tag{68}$$

Where:

- *ph'* is the conditional probability obtained from the history observed by the principal. Thus, the probabilities are updated period by period in the utility function of the principal.
- *ph''* is the conditional probability obtained from the history observed by the agents. Thus, the probabilities are updated periodically in the utility function of the agents.

All the types of agents observe the history in the same way.

- The relationship between the discount factors is: $\delta_p > \underline{\delta} > \bar{\delta}$.
- With $0 < \bar{\delta} < \underline{\delta} < \delta_p < 1$

Given that the principal needs to maximize utility, the first-order conditions must be defined. These are shown below:

$$u'\left(\underline{q}\right) = \frac{2-\delta_p-\underline{\delta}}{1-\underline{\delta}} e \tag{69}$$

$$u'\left(\underline{q}\right) = \underline{e} + \frac{1-\delta_p}{1-p} \left(\frac{(1-p)e}{1-\delta} + \frac{p\Delta e}{1-\delta_p} \right) = \frac{1-\delta_p}{1-\underline{\delta}} e + \frac{1-\delta_p}{1-\delta} \frac{p}{1-p} \Delta e \tag{70}$$

3.2.6 Numerical Example for the Learning by Repetition

This subsection describes the numerical case that demonstrates the impact of learning with labor incorporated into the repetition and decrease in the asymmetric information in the classical model for unobservable costs and effort levels.

3.2.6.1 Numerical Example for the Learning by the Repetition using Unobservable Costs

Using the same example explained in subsection, we have the following answers about the quantity produced when the decrease in asymmetric information by learning in the repeated game is considered. The relationship between the discount factor of the agents is $\delta_p > \underline{\delta} > \overline{\delta}$. Table 6 describes this new scenario for δ = 0.8 to the principal.

Table 6 - Production Level on the Learning by Repetition using Unobservable Costs

Agent	$\underline{\theta}$-Agent				$\overline{\theta}$-Agent			
	δ = 0.6				**δ = 0.2**			
	α = 1	**α =1.5**	**α = 2**	**α = 2.5**	**α = 1**	**α =1.5**	**α = 2**	**α = 2.5**
Payoffs	$700	$700	$700	$700	$300	$300	$300	$300
Production Level on t =1	100 units	40.25 units	16 units	6.26 units	30 units	8.53 units	2.4 units	0.66 units
Production Level on t >1	150 units	60.37 units	24 units	9.39 units	112.5 units	91.32 units	75 units	61.39 units

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To describe the numerical case of learning by repetition using unobservable costs, some hypotheses need to be presented.

1. The probability of agents appearing with high and low performance (***p*** and ***1-p***) is the same.
2. The costs of doing a certain activity are assumed by the optimal contract proposed in (64) and (65).

Given that the marginal cost of the efficient agent increased by 50% after the first 10 units were produced, and there is a difference between his discount factor $\underline{\delta}$ = 0.6 and δ_p = 0.8, namely, 0.2, the production levels can be analyzed using the optimal contract proposed as follows: $S'\left(\underline{q}\right) = \frac{2-\delta_p-\underline{\delta}}{1-\underline{\delta}}\theta^{\alpha}$.

So, $S'(\underline{q})=1{,}5\theta^{\alpha}$. Therefore, there is a gain in production when the repeated form is imposed, owing to the principal decrease in asymmetric information. Therefore, this will not produce 100 units, as in the classical model.

The new operation is the product of the $S'(\underline{q})=1{,}5\theta^{\alpha}$ and the ratio between monetary gains and associated costs for each agent in determining the task. $500/5^1 = 100$ units. As a result, 150 units can be produced, thereby showing the advantages of the principal in a play to repeated games, given that there is an increment of 50% of production for the efficient agent. Thus, the principal can reduce the initial proposal to reach his goal.

However, the learning with labor that was introduced into our model is presented and generates different results; for example, if α = 1,5, the efficient agent will produce another 60.37 units for the same value as the initial contract when compared with the nonlinear preferences by another 40.25 units with a growth of 49.98%. Moreover, there was a decrease of 32.92% when compared with the classical model. If α = 2, the high agent will produce another 24 units for the same value as the initial contract when compared with the nonlinear preferences by 16 more units with a growth of 50%.

In addition, there was a 70% decrease compared to the classical model. If α = 2,5, the agent will produce another 9.39 units for the same value as the initial contract when compared with the nonlinear preferences by another 6.26 units with a growth of 50%. In addition, there was a decrease of 86.58% compared to the classical model.

This also occurs with the inefficient agent, where the new operation uses $\frac{1-\delta_p}{1-\underline{\delta}}\theta+\frac{1-\delta_p}{1-\delta}\frac{p}{1-p}\Delta\theta$, taking into account the hypothesis of the same probability of occurrence for both agents (*p* and *1-p*). So, $S'(\underline{q})=0.25.\underline{\theta}+0.25.\Delta\underline{\theta}$.

Therefore, the new operation is the product of product $S'(\underline{q})=0.25.\underline{\theta}+0.25.\Delta\underline{\theta}$ and the ratio between monetary gains and associated costs for each agent in determining the task was $300/10^1 = 30$ units. Therefore, this will produce another 112.5 units if α = 1; 91.32 units if α = 1.5, 75 if α = 2 and 91.39 units if α = 2.5 showing, respectively. a percentage growth of: 275% if α = 1; 970.57% if α = 1.5; 3025% if α = 2 and 13746% if α = 2.5. This growth is high because of the large difference between the discount factors of each player.

3.2.6.2 Numerical Example for Learning by Repetition using Effort Level

Using the same example explained in Subsection A.3.2, we also have the following answers about the quantity produced when decreasing asymmetric information is considered by learning in the repeated game. Table 7 describes this new scenario for δ = 0,8 for the principal.

Table 7 - Production Level on Learning by Repetition using Effort Level

Agent	$\underline{\theta}$-Agent				$\bar{\theta}$-Agent			
	δ = 0.6				**δ = 0.2**			
	α = 1	**α =1.5**	**α = 2**	**α = 2.5**	**α = 1**	**α =1.5**	**α = 2**	**α = 2.5**
Payoff	$700	$700	$700	$700	$300	$300	$300	$300
Production Level on t =1	200 units	109.82 units	60 units	32.73 units	60 units	25.49 units	10.8 units	0.91 units
Production Level on t >1	300 units	164.73 units	90 units	49.09 units	75 units	53.66 units	39 units	28.48 units

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To describe the numerical case of learning by repetition using the effort level, some hypotheses need to be presented.

1. The probability of agents appearing with high and low performances (*p* and *1-p*) is the same;
2. The costs of doing a certain activity are supposed by the optimal contract proposed in proposed in (69) and (70).

Using the same operation rules for the Unobservable Costs case and que structure for (69) and (70), it can be seen that:

Learning with labor that was introduced into our model with effort level is presented and generates different results. For example, if α = 1.5, α = 2, and α = 2.5, the efficient agent will produce 50% more than the nonlinear model.

The inefficient agent will produce 25%, 110%, 261%, and 3029% more units, respectively, if α = 1, α = 1.5, α = 2, and α = 2.5.

3.3 LEARNING IN THE ECONOMIC CYCLES OF EMPLOYABILITY

This section was created to address the main topics related to variations related to the decision-making process of agents when they perceive that there are macroeconomic variations in employability cycles.

3.3.1 Learning Environments

As already discussed, learning is an inherent characteristic of rational individuals. Therefore, it is possible to learn in the most diverse types of scenarios for the most varied magnitudes and amplitudes. However, for the proper direction of the principal-agent learning model of economic cycles of employability, two possible scenarios (economic expansion) and (economic recession) will be defined and environmental information necessary for the achievement of the game will be extracted from them.

In the macroeconomic context, the values of economic indicators may deteriorate or improve. These indicators comprise a set of indices that determine the direction of the economy. If the values and/or indicators start to deteriorate in a generalized way, we characterize it as a period of economic recession. On the contrary, when these indicators jointly present more efficient performances, the scenario of economic expansion is characterized (Clamska & Klecka, 2020).

For the learning context in the principal-agent model, both scenarios are factors of interference in the model of preferences of contracted agents. In the first case, in an environment of economic expansion, the agents to be hired have more possibilities and offers, which causes their impatience rate and/or discount factor, which reflects this perception of the environment to become increasingly close to 1, which makes players more demanding to perform a certain task, causing the principal to review their disbursement concepts.

In economic recession scenarios, there are not so many job offers and agents have to accept the contract drawn up by the principal at lower values, since their impatience rate or discount factor is much lower than in the economic expansion scenario. More advantageous offers to the principal always occur in scenarios where the impatience rate decreases, due to the decrease in market supply. In this sense, there may be more profiles and rates of impatience, in addition to the classic ones previously mentioned. High performers are now subdivided into high performers and economic expansion and high performers and economic recession. On the other hand, low-performing agents are also divided into low-performing agents in the context of economic expansion and low-performing agents in the context of economic recession. All this is possible, as the discount factor has become a variable of perception and learning of the environment, as will be explained in the next topic.

3.3.2 Learning and Bayesian Games

When some players do not know the winnings of others, the game is commonly known as incomplete information. However, when players at the time of their move do not know the costs linked to certain activities or the behavior of others, it is called the imperfect information game. (Fudenberg & Tirole, 1994). In this sense, games in which there is information asymmetry can be characterized as Bayesian Games and be solved by the Bayesian Nash equilibrium.

Thus, according to Fudenberg & Tirole (1994), a Bayesian game is one that has imperfect information and that the equilibrium is characterized as a pair of strategies s1* and s2*, such that for each player e each possible value ci pegged strategy si maximizes the expected utility value of the game.

Bayesian games are games in which, at the beginning of the game, before players start planning their actions in the game, some players may already have some private information about the game that other players do not know. So often one can analyze situations where players currently have different private information that they have had for a long time, and it is unnatural to define the beginning of the game as being some point in the distant past before players read their private information. (Rego, 2014).

As the set of information present in the Bayesian game is imperfect for each agent, the concept of learning improves the level of information about the game and modifies the preferences of each agent and its respective result. Therefore, learning in a Bayesian game is necessarily vital to the achievement of the player's goals. The player (principal or agent) who has the largest amount of information will, consequently, have more competitive advantage in the formulation of the contract.

3.3.2.1 Markov Chains and Transition Probabilities

For the development of the learning model, non-stationary Markov models will be used in sub-topic 3.3, and for that, a brief definition of how the Markov processes are triggered is necessary. Thus, according to Norris (1998), it is a Markov process is a stochastic process where the probability distributions for its future development depend only on the present state, not taking into account how the process reached that state.

Markov processes are formally modeled by Markov models, which are state transition systems, where states are represented in terms of their probabilistic vectors, which can vary in time (discrete or continuous), and transitions between states are probabilistic. and depend only on the current state. If the state space is discrete (enumerable), then the Markov model is called a Markov chain.

The properties of these models are studied in terms of the properties of the state transition matrices that are used in their description. Markov processes can be characterized as stationary or non-stationary and this point defines a lot about how the learning model will be developed. To understand these concepts, it is necessary to discuss transition probabilities P(X) k + 1 = x k+1 |X)k(= xk } which therefore represent the probability of the state X k(+)1 be xk + 1 at time k + 1 given that state X(k) is xk at time k. If the Transition probabilities do not change with respect to time, they are said to be stationary. However, if there is variation in the Transition probabilities a Markov chain will be said to be non-stationary, so these probabilities change over time during each period.

3.3.2.2 Learning and Probabilities distributions

Probability distributions are used in this scenario to update transition matrices as the learning model assumes that players update their transition probabilities from observations. That is, when the transition matrix has uncertainty, a probability distribution is assumed that is updated by the players' perception. In this sense, a variation of the beta distribution, known as the Dirichlet distribution, is used.

3.3.2.2.1 Dirichlet Distribution

According to Bertucelli (2008), the transition matrix Π of the non-stationary Markov chain defined by Π ϵ R N x N given by

$$\Pi = \begin{bmatrix} \pi(1,1) & \pi(1,2)\dots & \pi(1,N) \\ \pi(2,1) & \pi(2,2)\dots & \pi(2,N) \\ \pi(N,1) & \pi(N,2\,)\dots & \pi(N,N) \end{bmatrix} \tag{71}$$

Where π(i,j) input is the probability that the transition to state j at time k + 1, given that state was i in the previous time step

$$\pi(i, j) = \Pr[x_{k+1} = j \mid x_k = i] \tag{72}$$

Note that $\sum \pi(i, j) = 1$. When the transition matrix Π is uncertain, we can take a fairly common Bayesian point of view Jaulmes (2005) and assume a previous Dirichlet distribution in each row of the transition matrix, and recursively update this distribution with observations

The Dirichlet distribution f_D at k time for a line N-dimensional transition model is given by pk = $[p_1, p_2, \dots, p_N]^T$ and hyperparameters (with $\alpha_i > 1$) $\alpha(k) = [\alpha_1, \alpha_2, \dots, \alpha_N]^T$, is defined as

$$f_D\left(p_k \mid \alpha(k)\right) = K\ \prod_{i=1}^{N} p_i^{\alpha_i - 1},\ \ \sum_I p_i = 1 = Kp_1^{\alpha_1 - 1} p_2^{\alpha_2 - 1} \dots\dots\left(1\ \sum_{i=1}^{N-1} p_i\right)^{\alpha_N - 1} \tag{73}$$

Where K is a normalization factor that guarantees the probability distribution integrates with unity. Each p_i is the nth column in the nth row, given that $p_i = \pi(m,i)$ and $0 \leq p_i \leq 1$ and $\sum p_i = 1$. Also, the hyperparameters α_i which can be interpreted as "counts", i.e. the times at which a given state transition was observed, easily updating the distribution based on new observations.

3.3.3 Learning and Employability Cycles

The cases shown above maintained the hypothesis that the actors involved did not suffer any influence from the economic scenario. When this hypothesis is relaxed, we have the so-called employability cycle.

Therefore, this game happens in an economic frame in which, in each period, there may be two possibilities. These economic scenarios are called employability cycles.

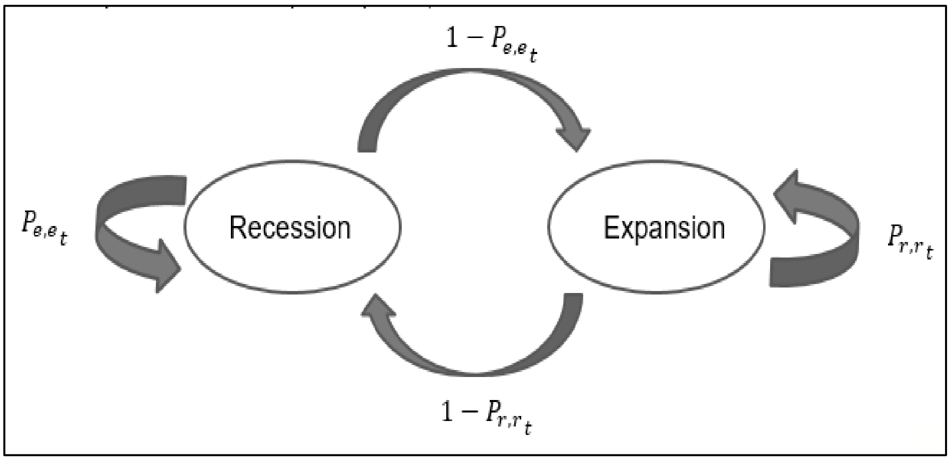
The first is an *economic expansion*, in which there are *plenty* of job offers, and it is easier for any agent to find jobs in one period and to be free to look for other possibilities in the next period. In this scenario, the bargaining power of the agents is greater than in other situations. These factors are *more critical* to the conditions and values offered by employers.

The other possibility is an *economic recession*, during which job vacancies drop to a few offers, and the probability of unemployment is higher.

In this case, bargaining power starts to decrease, and agents tend to *accept* the contracts offered by employers *faster* than normal.

The changes in the scenario were random and had unknown distributions. To describe this phenomenon in a specific period, we can use a Markov chain.

Figure 2 – The Cycles of Employability



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In these two possibilities, some characteristics can be found which are described below:

- The principal evaluates the actual scenario and contracts the agents.
- The agents evaluate the scenario and accept or refuse the proposal.

To understand how scenarios change, our model uses a finite non-stationary Markov chain that is described by a sequence of transition matrices defined in a common state space (in this study, two states, recession and expansion). In period k, the system moves from state i to j with probability $\Pi(k)_{ij}$. Therefore, the probability $\Pi(t)_{ij}$ changes over time during each period. Hence, the scenarios of this game can be represented by the transition matrix as follows:

$$\Pi_t = \left[P_{e,e_t} \; P_{e,r_t} \; P_{r,e_t} \; P_{r,r_t} \right] = \left[P_{e,e_t} \; 1\text{-}P_{e,e_t} \; 1\text{-}P_{r,r_t} \; P_{r,r_t} \right] \tag{74}$$

Therefore, players need to understand the scenarios and update their probabilities of accepting or refusing the formulation of the contract with these observations. To this end, we used the Dirichlet distribution in each row of the transition matrix. The use of this specific type of beta distribution is due to the possibility of changing the probabilities with each observation of the players, which leads to learning for the hiring logic.

The Dirichlet distributions for the first and second rows of the transition matrix were analyzed as follows:

$$D_{p_t} = \frac{\Gamma(a_1+a_2)}{\Gamma(a_1)\Gamma(a_2)} P_{e,e_t}^{\alpha_1\text{-}1} (1\text{-}P_{e,e_t})^{\alpha_2\text{-}1} \tag{75}$$

$$D_{p_t} = \frac{\Gamma(a_1+a_2)}{\Gamma(a_1)\Gamma(a_2)} (1\text{-}P_{r,r_t})^{\alpha_1\text{-}1} P_{r,r_t}^{\alpha_2\text{-}1} \tag{76}$$

Every period when we update the values of the probabilities of transitions, new information is generated about how likely periods of expansion and recession will be in the future. The expectation about the long-run probabilities of each state will be used to parameterize the discount factor.

From now on, the discount factor for all players is $\delta_i(\pi_e)_t$, thus reflecting the beliefs of player i in period t and their influence on the rate of impatience through successive observations.

Thus, the discount factor $\delta_i(\pi_e)_t$ is a variable that reflects the observation of the transition of scenarios of economic expansion and recession from a non-stationary Markov chain that updates the information of the players involved in the contractual relationship by using the Dirichlet distribution.

How does this expectation change the discount factor?

Two hypothesis are made:

1. Both types of agents benefit from a heated economy. Thus, if any agent deduces that by his observation in any given period that an expanding economy will frequently happen, his patience rate (discount factor) will increase, and both will be more critical.

2. The principal, on the other hand, can benefit from dealing with more brash agents as they can offer lower salaries due to their bad expectations of job vacancies in an economy in recession.

3. If an agent does not accept the contract, he or she will find a temporary job for one period.

This can be written as:

$$\frac{d\delta_p}{dp_e} < 0; \frac{d\underline{\delta}}{dp_e} > 0; \quad \frac{d\underline{\delta}}{dp_e} > 0 \tag{77}$$

The results above show that expansion periods are beneficial for the agents and prejudicial for the principal, given that the first-order conditions are positive for the employees (agents) and negative for the owner (principal).

In contrast, in a period of recession, the relationship between them is completely different, which is better for the owner (principal).

Proposition 1: Suppose that in a specific period τ, the discount factors are $\delta_p(\pi_e)_\tau$, $\underline{\delta}(\pi_e)_\tau$, and $\bar{\delta}(\pi_e)_\tau$, and every decision made in this period will use this discount factor to calculate its utility in the long run.

Therefore, if $t = \tau$, the highest and lowest utilities of agents can be described as follows:

$$\sum_1^T \underline{\delta}(\tau)^{t-1}(\underline{t} - \underline{\theta}\underline{q}) = \frac{1-\underline{\delta}(\tau)^T}{1-\underline{\delta}(\tau)}(\underline{t} - \underline{\theta}\underline{q}) \tag{78}$$

$$\sum_1^T \bar{\delta}(\tau)^{t-1}(\bar{t} - \bar{\theta}\bar{q}) = \frac{1-\bar{\delta}(\tau)^T}{1-\bar{\delta}(\tau)}(\bar{t} - \bar{\theta}\bar{q}) \tag{79}$$

Moreover, the principal maximizes his utility by the structure:

$$\text{Max } p\sum_1^T \delta_p(\tau)^{t-1}\left(S\left(\underline{q}\right) - \underline{\theta q}\right) + (1 - p)\sum_1^T \delta_p(\tau)^{t-1}\left(S(\overline{q}) - \overline{\theta q}\right) - p\sum_1^T \Delta\theta\overline{q} \quad (80)$$

Taking into consideration the following constraints:

- Participation Constraints

$$\sum_1^T \underline{\delta}(\tau)^{t-1}\left(\underline{t} - \underline{\theta q}\right) = \frac{1-\underline{\delta}(\tau)^T}{1-\underline{\delta}(\tau)}\left(\underline{t} - \underline{\theta q}\right) \geq 0 \quad (81)$$

$$\sum_1^T \overline{\delta}(\tau)^{t-1}(\overline{t} - \overline{\theta q}) = \frac{1-\overline{\delta}(\tau)^T}{1-\overline{\delta}(\tau)}\left(\overline{t} - \overline{\theta q}\right) \geq 0 \quad (82)$$

- Constraints on the Compatibility of Incentives

$$\sum_1^T \underline{\delta}(\tau)^{t-1}(\underline{t} - \underline{\theta q}) \geq \sum_1^T \overline{\delta}(\tau)^{t-1}(\overline{t} - \underline{\theta}\overline{q}) \quad (83)$$

$$\sum_1^T \overline{\delta}(\tau)^{t-1}(\overline{t} - \overline{\theta q}) \geq \sum_1^T \underline{\delta}(\tau)^{t-1}(\underline{t} - \overline{\theta}\underline{q}) \quad (84)$$

Maximizing the principal’s utility, we rewrite the second best formulated by Laffont & Martimort (2001) as follows:

$$S'\left(\underline{q}\right) = \underline{\theta} \quad (85)$$

$$S'(\overline{q}) = \overline{\theta} + \frac{p}{1-p}\frac{(1-\delta_p)(1-\underline{\delta}^T)}{(1-\underline{\delta})(1-\delta_p^T)}\Delta\theta \quad (86)$$

We can conclude that this payment to the high agent is lower than in the classical model for both scenarios, from which it can be concluded that learning with the changing scenarios of employability modifies the perception of the agents involved in the game.

$$\underline{\theta} + \frac{p}{1-p}\frac{(1-\delta_p)(1-\underline{\delta}^T)}{(1-\underline{\delta})(1-\delta_p^T)}\Delta\theta < \overline{\theta} + \frac{p}{1-p}\Delta\theta \quad (87)$$

The condition to accept the contract offered by the principal is described below, namely, if for a specific period τ, the utility in the long run obtained by observing a random scenario of the high-performance agent is lower than the gains obtained in the optimal contract, the contract is accepted.

$$\sum_1^T \underline{\delta}\left(p_{e_t}\right)^{t-1}\left(\underline{g} - \underline{\theta q}\right) < \frac{1-\underline{\delta}(\tau)^T}{1-\underline{\delta}(\tau)}\left(\underline{g} - \underline{\theta q}\right) \quad (88)$$

The decision-making problem for the low performance agent is analogous.

3.3.4 Analysis about the Learning in the Hiring Logic and Optimal Contracts

As can be seen, the changes arising from the insertion of the concept of learning and perception in all the agents involved in the game, for different scenarios and hypotheses, modify the optimal structuring classically defined, which brings differences in the results, bringing economic gains, and consequently, for the agents in each proposed situation, as can be better detailed in chapter 4 of this thesis.

4 THESIS CONCLUSIONS

Understanding the context in which the player is inserted has always been paramount for behavior in decision situations. In the logic of hiring and drafting optimal job contracts, classical models offer a preference structure of the agents involved in the game and advocate the importance of analyzing behavioral changes revealed by changes in these preference structures.

However, the preference structure, that is, each player's utility function, can be changeable and can behave according to each player's perceptions. Therefore, drawing up optimal contracts without considering the participants' learning in the most diverse areas, as shown in this thesis, becomes static and is not adaptable to the real and dynamic context that reflects the economic hiring scenario.

Therefore, by using the principal agent model of Laffont & Martimort (2001), changes in the optimality of the contract with increases and decreases in the values to be offered by the contractor can be verified.

In addition, there is the possibility of not contracting by simply understanding the current economic scenario, which shows that learning is continuous and that makes the formulation of static preference structures not adaptable to the current situation in the job market. Finally, an individual's preferences are functions of his/her continuous learning about the scenario, about the other agents, and about himself/herself regarding the object to be studied.

Thus, the main contributions of this work are to discuss the need to verify the degree of learning of all agents involved in the changes and influences of the environment in which they are inserted in the formulation of employment contracts and demonstrate the changes in the utility structures of each agent involved in understanding the environment in which the game is inserted and, consequently, the change in the cost-benefit ratio and the different optimal offers according to each scenario and degree of learning of the players.

In addition, the perception that the utility of work can vary is, in a way, one of the foundations and pillars of this research. When Laffont & Martimort (2001) glimpsed the hiring logic, they left a path very conducive to variations and different perceptions about how their utility would behave. Furthermore, with the contribution to the microeconomic theory of Jevons (1957), on the concepts of marginal disutility of work, the possibility of inserting non-linearity in the agency theory was well-known.

In this development, the research sought to find the most appropriate ways of inserting non-linearity into the utilities of the agents involved in the execution of the activity proposed by the principal. In the meantime, and understanding the needs of the agents involved in the game, the non-linear growths of production costs should be demonstrated in some way, for this reason, the non-linearity was treated exponentially in the agents' costs, which unfolded all the outcomes of this model now seen by the reader.

Therefore, the non-linearity proposed by this research, brought results that notably can be proven by the new optimality conditions inserted in the model, making the principal or delegate of activities have to better analyze the scenario and the preferences of each agent when developing activities to formulate the optimal work contract adapted to agents who understand or learn that their effort grows with each work activity performed.

The concept of repetition or sequentiality in the agency model brings out even more strongly that there is some learning involved in hiring relationships. This is because the players make their decisions in order to recognize the behavior of the other agents involved with each move. The game, in this sense, undergoes a constant updating of information by each player. It learns, reduces information asymmetry and only after that makes its decision.

The transformation into a repeated game proposed by Radner (1985) also brings a different view regarding the understanding of the scenario and strategic behavior due to the change in perceptions and, consequently, in their preferences. Now, the principal has the possibility to better understand each service provider and can formulate contracts with less disbursements and waste, which makes, so far, the most favorable model for those who delegate the activity, unlike the context of non-linearity where the marginal disutility of the work proposed in this work, gives greater advantages to the performers of the service.

Therefore, the repeated model of the agency brings with it one more extremely strong indication that without learning, the game is limited to the plastering of the optimal hiring without based on perceptions and learning so notably necessary for the current dynamic context.

The context of non-linearity, notably, would also be glimpsed in the repeated case of the agency model. In this sense, it can be observed that the asymmetries of information that were once reduced due to the transformation into a sequential game referred to in the model, have an impact on the gains in informational income earned by the agents due to the marginal disutility of work applied earlier in this study.

The intention was, exactly, to carry out the counterpoint between two factors that reflect strategic gains for opposite sides of the game. The activity delegate or principal gains from the sequential game, while service providers gain greater gains from the marginal disutility of work.

What can be observed was another adaptation of the classic hiring model to the most current and dynamic reality of perception of players from all sides of the game. As can be seen, the marginal disutility of work still brings losses to the principal, even and decreasing its information asymmetry with each move made in the sequential game. This demonstrates that the increase in information does not reduce the damage caused to the agent by each additional work unit performed and this is fully reflected in the results of Chapter 3 of this study.

The perceptions and learnings of the players involved in the agency theory model of contracting can also be shaped by macroeconomic developments as can be seen in Chapter 3 of this study. This time, the players have market information that influences their impatience rates or intertemporal discount factors, which brings a new aspect to the demonstration of the optimal contracting of the main agent model. Employability cycles alter the perception and preferences of players leaving them more or less susceptible to accepting proposals offered by the principal.

In this sense, it can be seen that the model is much more complex and comprehensive and that the form of hiring requires a prior analysis not only of the player and the context in isolation, but of an entire scenario so that, in this way, the hirings are beneficial to all those involved in the game.

In short, the labor and contracting markets suffer interference from a series of economic movements of a macro or micro scope that, in turn, interfere in the perceptions of the agents involved in the process of strategic interaction of contracting, which leads to changes and new interpretations in the concepts of optimal hiring already so widespread by classical agency theory, immersed in the microeconomic concepts of game theory.

Models and analysis tools such as those instituted by Laffont & Martimort (2001), known as the principal-agent model, were designed to interpret contracting scenarios and visualize the interferences that occurred in the medium to arrive at an optimal denominator for both agents.

Notably, the models carry with them the concepts of information asymmetry, since, as it is naturally a Bayesian game, that is, loaded with imperfect information, the hiring logic needs an increasingly comprehensive view and discussion in the literature, as can be seen from the review carried out by this study.

Information asymmetry leads to a loss of equity in the hiring process and it is in this gap that the models in this study were developed. The non-linear model incorporated non-linearity in costs and efforts unobservable by the contracting agent and this non-linearity, brought about by the concept of marginal disutility of work by Jevons (1957), brought optimal contracting results different from the classic model that is already so widespread. The principal or task delegate now needs to pay more to fill the space of gradually reducing utility by the time or amount of work to be performed.

When the same process of strategic interaction is shaped sequentially, it was noticed that the information asymmetry for both unobservable costs (adverse selection) and unobservable efforts (moral hazard) is increasingly smaller, which can lead to the most beneficial optimal result for the task delegate, that is, the principal.

These two concepts (non-linearity and repetition) were introduced and the results can be seen in chapter 3 of this study, bringing new incorporations to the already well-known agency literature.

Furthermore, learning agents increase their perceptions when the factor that translates their impatience in the repeated game becomes a temporal variable of perception and, in this last scenario, market perceptions also change the classic result of the main agent.

In short, the introduction of variation in perception and learning in the classic model of the agency makes it more up-to-date and consistent with the current context of variation in the decision-making rate, due to the intense variation of information present for each player involved, that is, the preferences are changing all the time, reflecting the set of information present at the time of decision making.

Therefore, the concept of dynamics of perception, utility and preferences of the players in the principal-agent model reflects, as can be seen in the work, the most reliable results with the reality of the current hiring logic.

4.1 LIMITATIONS

The main limitations of this thesis were about finding models that developed the non-linearity of unobservable costs and efforts and that brought with them the macroeconomic interferences of the markets that reflected this variation of perception in the hiring logic, which further sharpened the research to find ways to development of models that could reflect this reality. Another limitation that may be worth noting was the difficulty in finding statistical models that would describe the changes in perception necessary for the construction of the model in the utility function of the agents involved.

4.2 PROPOSAL FOR FUTURE WORK

Knowing that this model brought the concept of learning to the individuals involved in the hiring logic and that this interferes with the optimal results adapted to each scenario and that this learning was defined as constant throughout the construction of the model, it is suggested for future works such as first change the change in learning levels over time in repeated games, that is, to understand if more recent information has more impact on learning than more recent information.

Furthermore, with the introduction of the concept of non-linearity in the perceptions of the agents involved with the introduction of the exponential function α inserted in unobservable costs and efforts, that is, respectively, in models with adverse action selection and moral hazard. It is necessary to understand the thresholds that α, that is, in this study, this factor was considered fixed. However, when it varies continuously and tends to certain values, which can occur with optimal hiring optimality results. Therefore, it is suggested to analyze the continuity of the α factor and determine the optimal limits in the hiring logic.

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