**UNIVERSIDADE FEDERAL DE PERNAMBUCO**
**DEPARTAMENTO DE ENGENHARIA DE PRODUÇÃO**
PROGRAMA DE PÓS-GRADUAÇÃO EM ENGENHARIA DE PRODUÇÃO

# RISK MANAGEMENT IN COMPLEX PROJECTS USING DECISION THEORY

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Orientador: Fernando Menezes Campello de Souza, PhD.

RECIFE, FEVEREIRO/2012

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DISSERTAÇÃO SUBMETIDA À UFPE

PARA OBTENÇÃO DO GRAU DE MESTRE

POR

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***"RISK MANAGEMENT IN COMPLEX PROJECTS USING DECISION THEORY"***

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A comissão examinadora, composta pelos professores abaixo, sob a presidência do(a) primeiro(a), considera o candidato **MARCOS ANTONIO MARTINS DE ALMEIDA APROVADO.**

Recife, 16 de fevereiro de 2012.

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*I dedicate this dissertation to my family*

To every thing there is a season, and a time to every purpose under the heaven:

A time to be born, and a time to die; a time to plant, and a time to pluck up that which is planted;

A time to kill, and a time to heal; a time to break down, and a time to build up;

A time to weep, and a time to laugh; a time to mourn, and a time to dance;

A time to cast away stones, and a time to gather stones together; a time to embrace, and a time to refrain from embracing;

A time to get, and a time to lose; a time to keep, and a time to cast away;

A time to rend, and a time to sew; a time to keep silence, and a time to speak;

A time to love, and a time to hate; a time of war, and a time of peace.

[Ec 3:1-8]

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# RESUMO

O gerenciamento de projetos complexos em um mundo altamente competitivo, tornou-se um desafio para os decisores e executivos mundiais. O surgimento de inovações tecnológicas cada vez mais rápidas, juntamente com a velocidade das mudanças do mercado e restrições das mais diversas ordens, requerem cada vez mais do gestor, ações de conhecimento e maturidade para lidar com essas situações. O uso de técnicas de gestão e ferramentas adequadas, são requisitos necessários nos dias de hoje para vencer as dificuldades frente aos cenários que muitas vezes surgem diante da administração desses projetos, onde o objetivo principal é atingir e satisfazer os fatores de sucesso do projeto, condicionados e observados os atributos mínimos como: escopo, prazo, qualidade e custo.

Este trabalho propõe políticas e estratégias que se enquadram nas técnicas executivas de gestão, na identificação, monitoramento e medição dos riscos ao longo da implantação dos projetos em decisões sequenciais. Parâmetros são utilizados para o efetivo gerenciamento dos riscos com a incerteza sempre presente nos cenários considerados. As principais fontes de riscos serão analisadas. São utilizados constructos da Teoria da Decisão, como a edução e a análise da função utilidade de decisores dos projetos, além do cálculo dos riscos inerentes ao gerenciamento de projetos complexos. Métricas são propostas para a avaliação dos riscos na tomada de decisão ótima.

Serão consideradas a identificação e minimização dos impactos que possam surgir ao longo da execução dos projetos. Estudos de casos também serão analisados.

Palavras-chave: Teoria da Decisão, Riscos em Gerenciamento de Projetos, Estratégia de Gestão, Aplicação da Teoria dos Jogos.

# ABSTRACT

The management of complex projects in a highly competitive world has become a challenge for policy makers and executives worldwide. The development of technological innovations has increased very fast. With the speed of market changes and restrictions from various demands, managers are required to share their knowledge, have maturity to deal with these situations, manage techniques and use appropriate tools to overcome the difficulties they face. There are scenarios that often are in place before the administration of these projects has even begun. The main object of the present work is to identify and enable the success factors that can affect condition the qualities of scope, time, quality and cost.

This work proposes systematic strategies and techniques that fall under executive management that can identify, monitor and measure the risks of project implementation in sequential decision-making. It uses the tools of Decision Theory as a foundation for the use of risk management policies. Parameters are used for the effective management of risk with uncertainty always present in the scenarios considered. The main sources of risks are analyzed. Constructs are used in Decision Theory, such as the eduction and the analysis of the utility function of project managers and calculation of the risks inherent in managing complex projects. Metrics are proposed for risk assessment for correct decision making. Case studies have also been analyzed.

Keywords: Decision Theory, Risk Management in Complex Project, Strategy Management, Application of Game Theory.

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# 1 INTRODUCTION

## 1.1 Prologue

The management of complex projects requires that decision makers of an organization have the knowledge and use of scientific and technological tools to facilitate the management of those decisions.

Decision Theory, with its mathematical construct, is presented as an ally for the understanding and modeling in decision-proposed solutions for problems in the management of large projects. Similarly, the use of Game Theory in Economic Applications will be considered.

In general, decisions may be associated with the preparation of projects or in their implementation. Whatever the approach, the methodology used by Decision Theory applies and meets the requirements for making a successful project.

For example, in the execution of construction projects and complex engineering infrastructures, financial difficulties naturally arise. Execution time is another focus to be worked out depending on implementation standards of the project. These are often contrary to the interests of the customers and functional managers involved, and may at some point conflict with a process of tradeoffs between these attributes. One can meet the requirements of the project costs at the expense of quality or gain in execution time of the project and lose in its scope.

A profound knowledge of the variables and attributes involved in decisions of a project, is required on the part of the decision-maker (Leijten, 2010). If these sets of conflicting interests are not mapped out and fully explained, or if the tradeoffs implemented are not properly quantified and qualified decisions might be made that put the project in a less than optimal state. In this case, such conflicts can lead to non-compliance factors of project success (Hellwig, 2004). All of these aspects will be explained and exemplified in this dissertation.

## 1.2 Justification

The present work is based on experiences of management for the implementation of complex projects, in particular the case of implementation of the RandstadRail project (Leijten, 2010). Organizational and technical uncertainties combine with a lack of information experienced during the implementation of the project led to expensive results from both a financial and social standpoint and even resulted in serious accidents.

Another case of a large project refers to the management of a Public Private Partnership (Palma, 2009). Various forms of analysis are explored using tools such as the economic theory of principal-agent, according to Palma (2009). The inclusion of project characteristics and the generation of a risk matrix were decisive for an improved conduct of the project.

Both experiences led to the generation of this study.

## 1.3 Objective

The objective of this dissertation is to propose a means to systematize an optimal policy in the decision-making sequence for managing complex or large projects, based on a set of strategies, formalized using the mathematical principles of Decision Theory and the Theory of Games in Economic Applications.

**Details:**

An analysis will be made of the decision maker's utility function, with respect to this person's influence on the resulting solutions. Proper conduct of a project depends on the characteristics and profile of the decision-maker. This profile can be prone to risk, indifferent to risk or risk-averse. Greater gains can be obtained depending on the profile of the decision maker. In this case of greater gains, it can be said that the manager is an individual with characteristics for risk propensity. By running a lower risk, the manager may obtain an inadequate payoff in which case the manager would have the characteristic of risk aversion.

Two case studies of Decision Theory for eduction of managers of the utility function will be used as models: one for the management of a nuclear plant and the other for the implementation of a project for a museum of sacred art in the State of Pernambuco.

The next chapter will briefly discuss the basic constructs of Decision Theory and Game Theory.

## 1.4 Organization of Dissertation

The dissertation has seven chapters:

Chapter 1 is an Introduction;

Chapter 2 deals with the theoretical framework of Decision Theory and the Applications of Game Theory to Economics;

Chapter 3 presents the theory of uncertainty for a better understanding of this work;

Chapter 4 describes the main problems encountered in Complex Projects related to their organizational, business and technical natures;

In Chapter 5 presents solutions to complex projects using the Theory of Decision. It proposes strategies for minimizing project risks. A case study on the management of a nuclear power plant is analyzed;

In Chapter 6 a case study is presented for implementation of Risk Management for the implementation of a project for a Museum of Sacred Art in the State of Pernambuco;

Chapter 7 presents the conclusion of the results of the dissertation, with suggestions for new lines of research and future work.

# 2 THEORETICAL FRAMEWORK

Studies involving the management of complex systems usually require a structure with respect to information management. Whatever the level of decision making in an organizational structure of a project, there is a need for the existence of minimum information to support decision-making. In this sense, Decision Theory is structured based on axioms of the preference specification of the utility function of the decision maker. The needs and desires of the decision maker are primary in conflict situations according to Decision Theory, which is based on the theory of utility. These are fundamental for understanding the nature and consequences of a decision, including the associated risks.

When an individual, or group of individuals, who are decision-makers observe a given scenario and make a choice through an action, it will be done depending on their desire or preference for a particular benefit or payoff. The utility theory using mathematical constructs measures and quantifies this desire by using the utility function. For this the axioms of preference and construction of a utility function are used. This section presents a synthesized construction of mathematical Decision Theory, shown in detail according to Campello de Souza (2007b). It describes the sequence of the basic constructs of Game Theory in Economics Applications according to Osborne (2004).

## 2.1 Axioms of Preference

The strength of the axioms is the relation of preference based on a real function called the von Neumann & Morgenstern (1947) utility.

Let $A, B, C, ...... \in \mathcal{P}^*$

**Axiom 2.1.1** ***Completeness***: *$A \succsim B$ ("A is at least as desirable as B") or $B \succsim A$; this is equivalent to saying that or $A \succ B$ ("A is preferable to B"), or $B \sim A$("A is equivalent to B"), or $B \succ A$.*

**Axiom 2.1.2** ***Transitivity***:

*a) $A \succ B$ ("A is preferable to B") and $B \succsim C$ ("B is at least as desirable as C") $\Rightarrow A \succ C$ this implies ("A is preferable to C");*

*b)* $A \sim B$ *("A is equivalent to B") and* $B \sim C$ *("B is equivalent to C")* $\Rightarrow A \sim C$ *this implies ("A is equivalent to C").*

**Axiom 2.1.3** ***Dominance***:

*a)* *If* $A \succ B, \quad 1 \geq \lambda > 0$, *then for all* $C \in \mathcal{P}^*$ *has* $\lambda A + (1-\lambda)C \succ \lambda B + (1-\lambda)C$;

*b)* *If* $A \sim B, \quad 1 \geq \lambda > 0$, *then for all* $C \in \mathcal{P}^*$ *has*

$$\lambda A + (1-\lambda)C \sim \lambda B + (1-\lambda)C \tag{2.1.1}$$

It should be noticed that $\lambda A + (1-\lambda)B$ is an extensive form game, as shown in Figure 2.1 below:

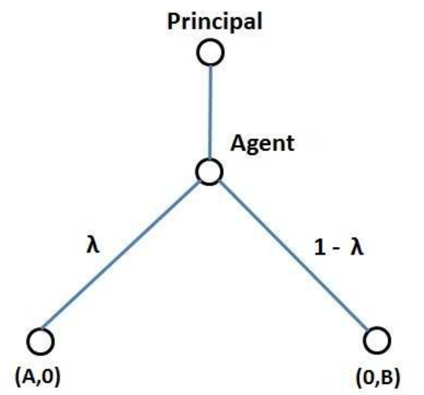


*Figure 2.1: Example of an Extensive Form Game.*

When, however, $\lambda A + (1-\lambda)B \in \mathcal{P}^*$ is a distribution that associates a probability to a *payoff* $p$. Then,

$$[\lambda A + (1-\lambda)B](p) = \lambda A(p) + (1-\lambda)B(p) \tag{2.1.2}$$

**Axiom 2.1.4** ***Archimedean***: *If* $A \succ B \succ C$, *then there are numbers* $\lambda$ *and* $\mu$ *so that* $1 > \lambda > \mu > 0$ *so that*

$$\lambda A + (1-\lambda)C \succ B \succ \mu A + (1-\mu)C \tag{2.1.3}$$

## 2.2 The Utility Function

The definition of the utility function, according to Campello de Souza (2007b) is given by:

**Definition 2.2.1** *: u is a utility function if:*

i) $u : \mathcal{P}^* \longmapsto \mathbb{R}$, for the entire distribution $A \in \mathcal{P}^*$ corresponds a real number $u(A)$.

ii) The order must be matched. If $A \succsim B \Leftrightarrow u(A) \geq u(B)$, reflecting the order of preference.

iii) The utility assigned to a convex combination of distributions is just a convex combination of distributions utilities, ie:

$$u[\lambda A + (1-\lambda)B] = \lambda u(A) + (1-\lambda)u(B) \tag{2.2.1}$$

The theorem which assures the existence of the utility function arises from the conjunction of the four axioms of preference as described in section 2.1:

1. **Lemma 2.2.1** *Monotonicity:*

   *If* $A \succ B$, $\lambda \geq \mu$, *then* $\lambda A + (1-\lambda)B \succ \mu A + (1-\mu)B$.

2. **Lemma 2.2.2** *Uniqueness:*

   *If* $A \succ B$ *and* $\lambda A + (1-\lambda)B \sim \mu A + (1-\mu)B$, *then* $\lambda = \mu$.

3. **Lemma 2.2.3** *Representation:*

   *If* $A \succsim B$ *if only if* $u_{\underline{P},\overline{P}}(A) \geq u_{\underline{P},\overline{P}}(B)$.

   *where:* $u_{\underline{P},\overline{P}}(A) = sup\{\lambda : A \succ \overline{B} + (1-\lambda)\underline{A}\}$, $\underline{P}$ *is the least desirable distribution in the consequences set and* $\overline{P}$ *is the most desirable.*

4. **Lemma 2.2.4** *Linearity:*

   $u[\lambda A + (1-\lambda)B] = \lambda u(A) + (1-\lambda)u(B)$.

5. **Lemma 2.2.5** *Extension: If $u$ is a utility function, then $u^* = au + b$, $a > 0$ , is also a utility function.*

   *where:*

   $b = -au_{\underline{\underline{P}},\overline{\overline{P}}}(\underline{P})$ *and*

   $a = \frac{1}{u_{\underline{\underline{P}},\overline{\overline{P}}}(\overline{P}) - u_{\underline{\underline{P}},\overline{\overline{P}}}(\underline{P})}$, *in conformity with*

   $u_{\underline{\underline{P}},\overline{\overline{P}}}(P) = sup\{\lambda : P \succ \lambda\overline{\overline{P}} + (1-\lambda)\underline{\underline{P}}\}$.

The demonstrations of the lemmas are found in Campello de Souza (2007b).

### 2.2.1 A Measure of Risk Aversion

**Definition 2.2.2** *: Due to the curvature of the utility function, one can define local risk aversion at $p$, denoted by $r(p)$, as:*

$$r(p) = -\frac{\frac{d^2u}{dp^2}}{\frac{du}{dp}} \tag{2.2.2}$$

All features with respect to $u$ are preserved in the role of risk aversion.

The function $r(p)$ locally measures the curvature of the utility function. In the region where the utility function is convex, the decision maker is willing to take risks in the observed range. If the region is concave, the decision-maker is risk averse. If the utility function is linear, the decision maker is indifferent to risk.

The matrix of absolute risk aversion can be calculated as follows:

$$R(p) = [r_{ij}] = -\left[\frac{u_{ij}(p)}{u_i(p)}\right] = -\left[\frac{\frac{\partial u^2}{\partial p_i p_j}}{\frac{\partial u}{\partial p_i}}\right] \tag{2.2.3}$$

### 2.2.2 The Basic Structure of Decision Theory

Decision Theory's fundamental construct is to carry out project risk assessment.

A) **State of Nature**

The states of nature is represented in classical form, for $\Theta$, and is the object which the decision-maker is interested in accordance with:

$\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, where $\Theta$ can represent the finite, infinite, discrete or continuous.

The elements of the set $\Theta$, characterize all possible outcomes in which nature presents itself, generating a number of scenarios that will influence the decision-making process.

B) **Action Set**

The set of all actions is represented by $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$ and defines a set of options which lead to a purchase or payoff according to the decision-maker's choice.

C) **Goods or Payoffs Set**

Once the decision-maker chooses a path for a manager's action, he will have as a result access to goods or payoffs, which are represented by the set $\mathcal{P}$.

$\mathcal{P} = \{p_1, p_2, \ldots, p_l\}$. The set $\mathcal{P}$ can be modeled by a column vector, which would be represented by:

$$\vec{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_l \end{bmatrix} \tag{2.2.4}$$

In this case the number of possible comparisons are many, making a decision difficult even though an eduction process is go on.

D) **Observations Set**

The set of observations represents the universe that the decision-maker has to try to infer from the state of nature, since $\{\theta\}$ generally can not be read or measured directly. The representation of this set is given by $\mathcal{X} = \{x_1, x_2, \ldots, x_t\}$.

E) **Probabilistic Mechanisms**

Managers usually have difficulty in managing all aspects and details when monitoring the implementation of big projects. The situation becomes more complex when uncertainties are present in the technical and organizational processes inherent to projects. The uncertainty is directly related to the probabilistic mechanisms. Among these mechanisms are, for example, the consequence function and the likelihood function and prior distribution of the states of nature.

i) **Consequence Function**

When a decision maker takes a certain action $a_i$ which is available in the set $\mathcal{A}$ =$\{a_1, a_2, ...., a_m\}$, a probabilistic mechanism will result in response to the decision-maker, which will depend always on the state of nature, represented by the function $\theta_\mathbf{i}$, and the action $a_i$ taken by the decision-maker. The function result can be formed from a set of all families of probability distributions for all goods, as represented by the set $\mathcal{P}^* = \{\text{P}\}$, where the probability distributions $P$ are called the consequence function, denoted by the representation $P(p|\theta, a)$. In the continuous case the representation is given by $F_{\mathcal{P}|\theta,\mathcal{A}}(p|\Theta, a)$.

ii) **The Likelihood Function**

Before the project manager acts, the decision-maker usually observes a variable that holds a relationship with the states of nature and forms part of the set of observations $\mathcal{X} = \{x_1, x_2, \ldots, x_t\}$, using a probability distribution represented by $P(x|\theta)$, which represents a likelihood function.

iii) **The Prior Distribution on States of Nature**

The uncertainty of probabilistic nature, studied as part of the objectives of this work as the construction of the probability distribution of $\pi(\theta)$ could come from two sources:

a) A set of data, collected over time and stored in a database, or

b) The knowledge of an expert or group of experts on the subject.

In any of the sources, the resulting prior distribution over the states of nature, denoted by $\pi(\theta)$, represents a fundamental piece of information for knowledge about the result of the function $P(p|\theta, a)$ ie, as the decision maker takes action $a$, nature randomly selects state $\theta$, which consequently generates a family of distribution $P(p|\theta, a)$ on the payoffs.

Expert knowledge may influence the determination of the probability distribution to over state nature, according to Sousa Júnior (2004).

### 2.2.3 Decision Making-Rules

A rule for decision-makers is a procedure that allows the manager to select an action from the set of actions available in $\mathcal{A}$ =$\{a\}$ and represented by:

$$d : \underset{x \longmapsto d(x)=a}{\mathcal{X} \rightarrow \mathcal{A}} \tag{2.2.5}$$

Thus, the decision making rule is a function, in which the domain is represented by the set of observations $\mathcal{X}$ =$\{x\}$. The set of possible decision rules is represented by $\mathcal{D}$ = $\{d\}$.

If the number of actions and observations are finite numbers, the possibilities of alternative decisions are

$$||\mathcal{D}|| = ||\mathcal{A}||^{||\mathcal{X}||} \tag{2.2.6}$$

A) **Loss Function**

The loss function $L(\theta, a)$ represents the damage or losses incurred by the decision-maker when taking a decision $a$, the result of an observation $x$, and when nature chooses a state $\theta$. The mathematical representation is that the loss is the negative of the utility:

$$L(\theta, d(x)) = -u(P(p|\theta, d(x))) \tag{2.2.7}$$

B) **Risk Function**

The oldest known citation on the use of risk for decision making is contained in the Talmud, the sacred book written by Jewish Rabbis between the years 200 and 500 AD. In this book there was an argument about the legitimacy or not of a man separated from his wife, because of a suspicion that she might have had another man before marriage. The rabbis prepared their response based on the set of possible alternatives (whether or there had been relationships with her husband or another man). Based on these possibilities it could be established that the husband should not be separated from his wife, because the odds in his favor were fewer than the wife's. The calculation of probabilities did not exist.

There are several versions of the origin of the word *risk*. One of the records dating back to the fourteenth century, cites the Castilian *riesgo*, but still does not have the connotation of potential danger. The etymology of the word *risk* suggests that it originated from the Latin *resecum*, which was used to describe situations related to sea voyages, such as a hidden danger at sea. The concept of possibility also appears at this point.

The notion of risk in systems of probabilistic risk assessment is defined as a potential harmful result inherent in an activity or action. The risk is characterized by two quantities according to Cristino (2007):

i) the magnitude (or severity) of adverse consequences that may result in the potential of a particular activity or action.

ii) the probability of occurrence of certain adverse consequences.

Risk function is defined as the expected loss function, given the probability distribution $P(x|\theta)$:

$$R_d(\theta) = \sum_{x \in \mathcal{X}} L(\theta, d(x))P(x|\theta)$$
$$R_d(\theta) = E(L|\theta, d) = E_\theta[(L(\theta, d(x))] \quad (2.2.8)$$

Risk is the expected value of loss when nature chooses $\theta$ and the decision-maker uses a decision rule $d$.

C) **Bayes Risk**

This is an unknown distribution $P(x|\theta)$, but with information from the distribution $\pi(\theta)$ taken from information experts, can calculate the risk for a given action $a = d(x)$ using the prior distribution:

$$R_d = \sum_{\theta \in \Theta} L(\theta, d)\pi(\theta) \quad (2.2.9)$$

To calculate the risk of a decision rule, it is necessary to have the probability distribution of states of nature $\pi(\theta)$.

For a decision choice rule, Bayes risk can be calculated as:

$$r_d = -u(P(p|d)) = -\sum_{\theta \in \Theta} \pi(\theta)u(P(p|\theta, d)) = \sum_{\theta \in \Theta} \pi(\theta)R_d(\theta) \quad (2.2.10)$$

The best decision rule is chosen by minimizing $r_d$, and the variable choice of a deterministic decision rule $d$. Mathematically it would be:

$$\min_{\{d\}}(r_d) = \sum_{\theta \in \Theta} \pi(\theta)R_d(\theta) \quad (2.2.11)$$

### 2.2.4 Methods of Eduction for the Utility Function

The eduction of the utility function is to build a set of relevant characteristics for decision-makers. These characteristics can be identified, measured and combined in a series of attributes that partly characterize the psychology of the decision-makers. These features come into play in a subjective way as required in situations where the choice of an action in made by a decision-maker. The decision usually comes through an evaluation of possible alternatives, an estimate of the consequences, and finally the selection of the most appropriate action.

To survey the decision maker's utility function, when he is the project manager, two methods can be used as described by Moraes (2003) and Bezerra (2003), namely:

A) The Method of Superimposed Tracks uses the technique of confrontation between a good or a correct value and lottery or gambling. The indifference of the decision-maker will be explored trough an eduction protocol and a series of specific questions to perceive the value of $\lambda$, where the individual is indifferent, ie:

$$P \sim \lambda \overline{P} + (1-\lambda)\underline{P} \tag{2.2.12}$$

At the end of the series of questions and different game situations, obtaining the varieties of $\lambda$, we can construct the curve of the decision maker's utility function. In cases where the values of $\overline{P}$ and $\underline{P}$ are far apart, there is a greater difficulty to award an $\lambda$, in this case, the value of $\lambda$ must be allocated by tracks. Then, using the property that the relationship between the different preferences are invariant with, related to the scale used, interval scales are provided, according to Campello de Souza (2007b).

For the eduction of the utility function, the Superimposed Tracks method is used in this dissertation as implemented in System Preferences Eduction - SEP, developed by Albuquerque (2011).

B) In the second method, the decision-maker chooses between two games. This method requires a greater consistency on the part of the individual. In the eduction process it is called Linear Programming, which can be presented as follows, according to Campello de Souza (2007a):

$$\max_{\{u_i\}} \sum_{j=1}^{n} (n-j+1)u_j \tag{2.2.13}$$

subject to

$u(G_i) - u(G_l) \geq 0$ ou $u(G_i) - u(G_l) \leq 0$, depending on the decision maker.

$u_{\underline{p}} = 0$

$u_{\overline{p}} = 1$

$u_{n-1} - u_n \leq 0$, condition of the utility function is monotonic.

where:

$u_j = u(p_j)$,

$\overline{p} =$ melhor $p_j$,

$\underline{p} =$ pior $p_j$,

$G_i = \lambda_i p_j + (1 - \lambda_i) p_k$, and

$G_l = \lambda_l p_m + (1 - \lambda_l) p_r$.

### 2.2.5 Operational Systematics of Decision Theory

Figure 2.2 shows the main elements of Decision Theory, architecture, and interconnection operations.

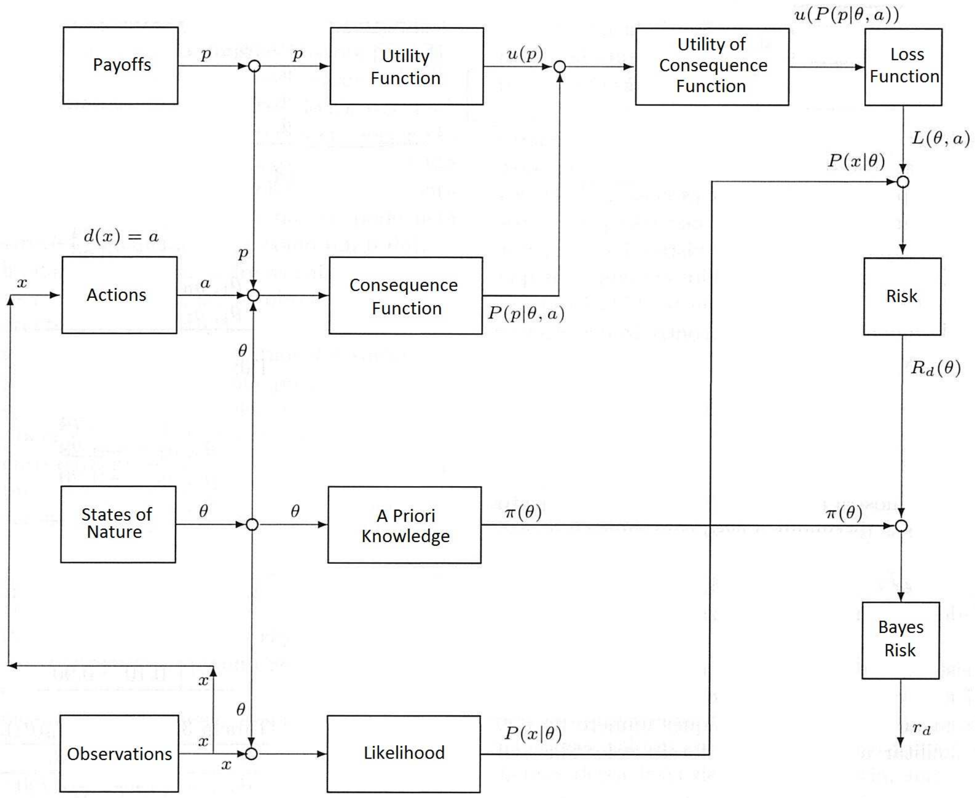


*Figure 2.2: Architecture and Operation of Decision Theory (Campello de Souza, 2007b).*

## 2.2.6 Profiles of Decision Makers from the Utility Function

To analyze the profiles of decision-makers, it is necessary to have the decision maker's utility function. Considering the Weierstrass approximation theorem, which ensures that every continuous function defined in a closed and bounded interval can be uniformly approximated by polynomials. From this premise, a case study can be formulated from the representation of the utility function using an expression formed by polynomials. Thus, the utility function is a quadratic expression, taken as an example in Campello de Souza (2012) with the parameters defined as $k_1$ and $k_2$:

$$u(p) = k_1 p + k_2 p^2 \tag{2.2.14}$$

$k_1$ and $k_2$ in practice, would be the parameters associated with the psychological characteristics of the decision-makers.

Since $u(1) = 1$ and $u(p)$ must be monotonically non-decreasing in $p$, we have:

$$u(1) = k_1 + k_2 = 1, \text{ and}$$

$$\frac{du}{dp} = k_1 + 2k_2 p \geq 0, \forall p.$$

Solving the system of equations, we have:

$$0 \leq k_1 \leq 2 \text{ e } 1 \leq k_2 \geq -1$$

Taking the first derivative, a point of maximum or minimum can be determined:

$$\frac{du}{dp} = k_1 + 2k_2 p = 0 \therefore p^* = \frac{k_1}{2k_2} = \frac{1 - k_2}{2k_2}$$

The calculation of the second derivative shows whether the utility function will have a maximum or minimum point, depending on the assumed values of the parameter $k_2$:

$$\frac{d^2u}{dp^2} = 2k_2 \tag{2.2.15}$$

The risk aversion function, given using equation 2.2.2, when applied to the utility function in equation 2.2.14 results in:

$$r(p) = -\frac{\frac{d^2u}{dp^2}}{\frac{du}{dp}} = -\frac{2k_2}{k_1 + 2k_2 p} \tag{2.2.16}$$

A) **Risk Averse Decision-Makers**

Depending on the characteristics of individuals, decision-makers have a utility function described by a concave shape, and exhibit attributes that lead to features with risk aversion, as shown by the following equations:

The region of concavity happens when $1 < k_1 \leq 2$ and $-1 \leq k_2 < 0$, and increased risk aversion has the maximum concavity when:

$$u(p) = 2p - p^2 \tag{2.2.17}$$

so,

$$\frac{du}{dp} = 2 - 2p \tag{2.2.18}$$

and the function of risk aversion, is then:

$$r(p) = \frac{2}{2 - 2p} \tag{2.2.19}$$

The curve of the utility function when the decision maker has an extreme aversion to risk is shown in figure 2.3.

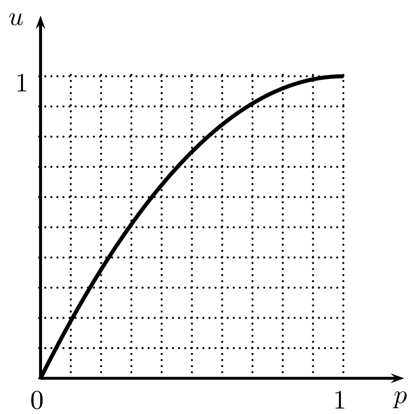


*Figure 2.3: Graphic of the Utility Function when the Decision-Maker is Risk Averse (Campello de Souza, 2012).*

B) **Decision-Makers Indifferent to Risk**

In this case, decision makers have a utility function with a linear curve with respect to goods or payoffs. There is an indifference to the risks associated in making the decision.

The region where the curve is linear happens when $k_1 = 0$ and $k_2 = 0$, is shown in the equation of the utility function:

$$u(p) = p \tag{2.2.20}$$

so,

$$\frac{du}{dp} = 1 \tag{2.2.21}$$

and the function of risk aversion, is then:

$$r(p) = 0 \tag{2.2.22}$$

The curve of the utility function when the decision maker is indifferent to risk is shown in figure 2.4.

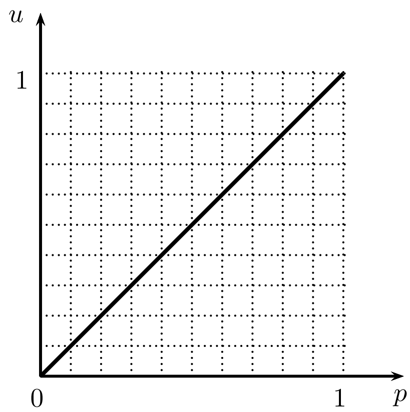


*Figure 2.4: Graph of the Utility Function when the Decision-Maker is Indifferent to Risk (Campello de Souza, 2012).*

C) **Decision-Makers Propensity to Risk**

Decision-makers who exhibit a utility function with a convex shape, incorporate attributes that have features prone to risk, according to the following equations:

The region of convexity arises when $0 \leq k_1 < 1$ e $0 < k_2 \leq 1$

Thus, the maximum convexity of the utility function is presented with:

$$u(p) = p^2 \tag{2.2.23}$$

so,

$$\frac{du}{dp} = 2p \tag{2.2.24}$$

and the function of risk aversion, is then:

$$r(p) = -\frac{1}{p} \tag{2.2.25}$$

The curve of the utility function when the decision maker has the highest propensity to risk is shown in figure 2.5.

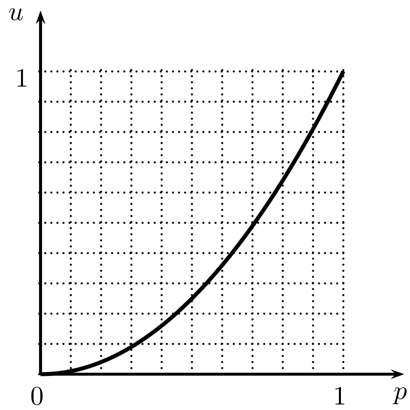


*Figure 2.5: Graph of the Utility Function when the Decision-Maker is Propensity to Risk (Campello de Souza, 2012).*

## 2.2.7 When the Utility Function Is Dependent on Payoffs

A) The Utility Function with Two Attributes

When a utility function, called quadratic form in $\mathbb{R}^2$, is given, it is an attribute function $(p_1, p_2)$, with real coefficients, defined by the parameters $k_i, i = 1, 2, ...,6$, where $k_i$, in practice, would be parameter associated with the psychological characteristics of decision-makers.

$$u(p_1, p_2) = k_1 p_1^2 + k_2 p_2^2 + k_3 p_1 p_2 + k_4 p_1 + k_5 p_2 + k_6 \tag{2.2.26}$$

Rewriting $u(p_1, p_2)$ in matrix notation, we have:

$$u(p_1, p_2) = \begin{bmatrix} p_1 & p_2 \end{bmatrix} \begin{bmatrix} k_1 & k_3 \\ k_3 & k_2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} k_4 & k_5 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + k_6 \tag{2.2.27}$$

Calculating using the eigenvalues $\lambda_1$ and $\lambda_2$ the linear operator represented by the matrix is:

$$[A] = \begin{bmatrix} k_1 & k_3 \\ k_3 & k_2 \end{bmatrix} \tag{2.2.28}$$

and with the orthogonal unitary eigenvectors $\vec{u_1} = (x_{11}, x_{21})$ and $\vec{u_2} = (x_{12}, x_{22})$, we obtain the matrix of base change $[I]^{\beta}_{\alpha}$, in order to obtain the following rotation:

$$u(p'_1, p'_2) = \begin{bmatrix} p'_1 & p'_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} p'_1 \\ p'_2 \end{bmatrix} + \begin{bmatrix} k_4 & k_5 \end{bmatrix} \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{bmatrix} \begin{bmatrix} p'_1 \\ p'_2 \end{bmatrix} + k_6 \tag{2.2.29}$$

The expression 2.2.29 is the canonical matrix form expression 2.2.26, or:

$$u(p_1, p_2) = u(p'_1, p'_2) = \lambda_1 p'^2_1 + \lambda_2 p'^2_2 + k_7 p'_1 + k_8 p'_2 + k_6 = 0 \tag{2.2.30}$$

which is the equation of the conic given in equation 2.2.26, but referenced to the system $p_1 0 p_2$. The classification of conic curves is defined by its eigenvalues, as follows:

1. If $det[A] = \lambda_1 \times \lambda_2 > 0$ then the conic is represented by an ellipse or some degeneration (point or empty).
2. If $det[A] = \lambda_1 \times \lambda_2 = 0$ then the conic is represented by a parabola or some degeneration (point, empty or pair of parallel lines).
3. If $det[A] = \lambda_1 \times \lambda_2 < 0$ then the conic is represented by a hyperbola, or some degeneration (pair of intersecting lines).

Since $u(\overline{p_1}, \overline{p_2}) = 1$ and $u(p_1, p_2)$ must be monotonically non-decreasing in $p$, we have:

$$u(1,1) = k_1 + k_2 + k_3 + k_4 + k_5 + k_6 = 1 \text{ and,}$$

$$u(\underline{p_1}, \underline{p_2}) = 0, \text{ thus, } u(0,0) = 0, \text{ thus, } k_6 = 0$$

Furthermore, $\frac{\partial u}{\partial p_1} \geq 0$ and $\frac{\partial u}{\partial p_2} \geq 0$, so

$$\frac{\partial u}{\partial p_1} = 2k_1p_1 + k_3p_2 + k_4 \geq 0$$

and,

$$\frac{\partial u}{\partial p_2} = 2k_2p_2 + k_3p_1 + k_5 \geq 0$$

On the other hand, the indifference curve in the total differential of $u(p_1, p_2)$ must be zero

$$\partial u(p_1, p_2) = \frac{\partial u}{\partial p_1} dp_1 + \frac{\partial u}{\partial p_2} dp_2 = 0$$

Thus, the Marginal Rate of Substitution (MRS), then can be calculated:

$$MRS = -\frac{dp_2}{dp_1} = \frac{\frac{\partial u}{\partial p_1}}{\frac{\partial u}{\partial p_2}} = \frac{2k_1p_1 + k_3p_2 + k_4}{2k_2p_2 + k_3p_1 + k_5} \tag{2.2.31}$$

The matrix of risk aversion according to the equation 2.2.6 is:

$$R = [r_{ij}] = -\frac{\frac{\partial u^2}{\partial p_1 \partial p_2}}{\frac{\partial u}{\partial p_1}} = -\frac{k_3}{2k_2p_2 + k_3p_1 + k_5} \tag{2.2.32}$$

Note that the parameter $k_3$ in equation 2.2.26, specific to each decision-maker weighs on the crossed attributes $p_1p_2$ emerging from a relationship of mutual dependence of the attribute $p_1$ on the attribute $p_2$ and vice-versa. The parameter $k_3$ appears in the secondary diagonal of the matrix in equation 2.2.27, in addition to appearing

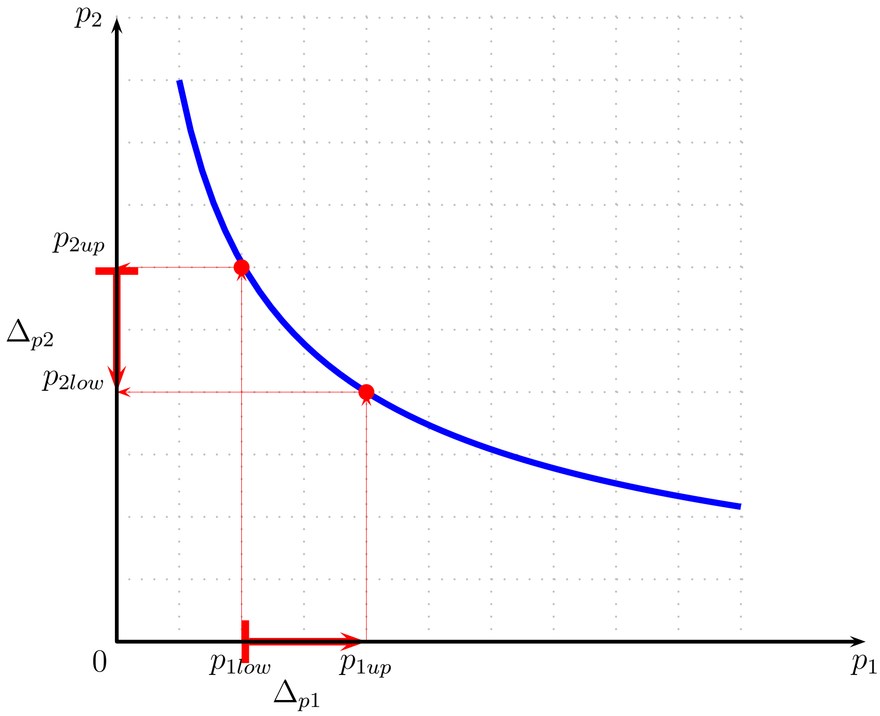


*Figure 2.6: Example of Marginal Rate of Substitution between Goods.*

in the equation of the Marginal Rate of Substitution in equation 2.2.31, and the array of risk aversion in equation 2.2.32. A special case where the utility function suggests an independence among the attributes can be seen in Alencar (2010) and Brito (2010).

**Tradeoffs between the Attributes**

In the expression of MRS, equation 2.2.31, depending on the values of $k_i$, the indifference curve of the attributes may be more or less pronounced, implying thereby that the exchange ratio between the attributes can be significant, i.e., a small variation in $p_1$, can reflect on a great loss at $p_2$, in a decision in which earned attributes in the amount of $\Delta_{p1}$, can be lost to the magnitude $\Delta_{p2}$, and vice versa, as shown in figure 2.6.

The utility function of the decision-maker is at the core of Decision Theory, mainly in project management. The possible consequences, which may be gain, loss, or indifference with respect to the payoffs, reflect the preference of the project management. Thus the parameters of the utility function establish an association between the payoffs and the actions of the project manager.

In Decision Theory, it is natural that there be an escalation in the order of preferences in terms of consequences for the decision. This ordering can be formed into a range of best result payoff. Thus if sixteen payoffs are considered, representing 100% for the worst - 0%, we have:

$$\vec{p}^{\,1} \prec \vec{p}^{\,2} \prec \vec{p}^{\,3} \prec \cdots \prec \vec{p}^{\,16}$$

B) The Utility Function with Three Attributes

When a utility function, called a quadratic form in $\mathbb{R}^3$, is given, it is an attribute function $p_1, p_2, p_3$, with real coefficients, defined by the parameters $k_i, i = 1, 2, \ldots, 10$, where $k_i$, in practice, would be parameter associated with psychological characteristics of the decision makers.

$$u(p_1, p_2, p_3) = k_1p_1^2 + k_2p_2^2 + k_3p_3^2 + k_4p_1p_2 + k_5p_1p_3 + k_6p_2p_3 + k_7p_1 + k_8p_2 + k_9p_3 + k_{10} \tag{2.2.33}$$

As seen in the case of the quadratic form in $\mathbb{R}^2$, it is possible to reduce a quadratic form in $\mathbb{R}^3$ to a canonical form:

$$\begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} \begin{bmatrix} k_1 & k_4 & k_5 \\ k_4 & k_2 & k_6 \\ k_6 & k_6 & k_3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} p_1' & p_2' & p_3' \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} p_1' \\ p_2' \\ p_3' \end{bmatrix} \tag{2.2.34}$$

The form

$$u(p_1', p_2', p_3') = \lambda_1 p_1'^2 + \lambda_2 p_2'^2 + \lambda_3 p_3'^2 \tag{2.2.35}$$

is called the canonical form of the quadratic form in $\mathbb{R}^3$ or diagonalized quadratic form.

Thus, the quadrics are formed by the set of points in $\mathbb{R}^3$, whose coordinates $p_1$,$p_2$,$p_3$ against the canonical base satisfy the equation 2.2.33.

An example of a quadratic function in $\mathbb{R}^3$ is given in Figure 2.7.

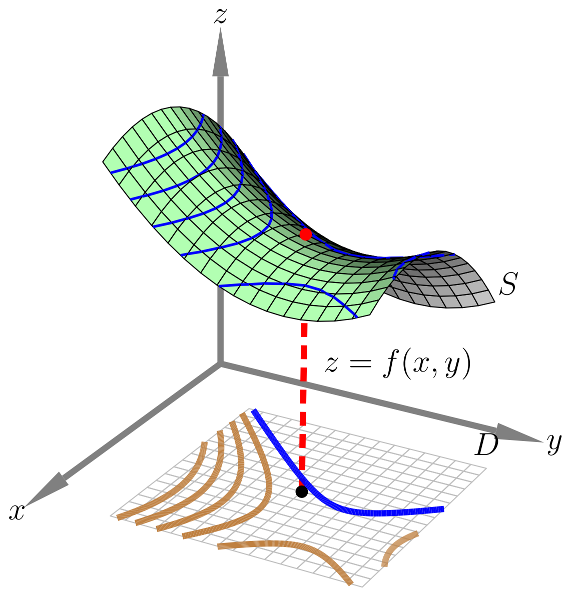


*Figure 2.7: Example of a Quadratic Function in* $\mathbb{R}^3$.

The function $z = f(x,y) = [(y-5)^2-(x-5)^2]/6+5$, the blue curve, shows the outline of the plan in the $xy$ range of $z = [4.9, 6.5]$.

As an example, suppose that the coordinate $z$ represents attribute $p_3$ indicating levels of project quality, the coordinate $y$, the representation of attribute $p_1$ indicating the project cost, the coordinate $x$, the representation of attribute $p_2$ indicating the time of execution.

Thus, for a given value of $z = 5$, in the range $[4.9, 6.5]$, as shown in Figure 2.7, a specific level of project quality is indicated. This value generates a projection on the plane $xy$ through the function $f(x, y)$, representing the attibutes $p_1$ and $p_2$, ie for a given level of quality in the project, there is a tradeoff between cost and runtime of the project. For another level of quality $z = 6$, other conditions of tradeoffs between cost and time are presented. It will be the project manager, within his profile, who is the one to make the decision as he sees fit, as shown in the example in Figure 2.7. With the projection on the plane $xy$, the analysis to be performed is similar to the tradeoff shown in Figure 2.6.

One truth is that the value of the utility of attribute and risks are associated. This functional relationship was predicted by von Neumann and Morgenstern (1947) when they generated a preference structure.

Extending to a more general case, the utility function $\mathbb{R}^n$ generates a set of pairs of uncountable $p_i p_j$ tradeoffs, when designed in two-dimensional planes. As in the real world, complex projects have a set of payoffs $\mathcal{P} = \{p_1, p_2, \ldots, p_l\}$. The set $\mathcal{P}$ can be modeled by a variable vector, which would be represented by:

$$\vec{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_l \end{bmatrix} \tag{2.2.36}$$

Where each coordinate of the vector $\mathcal{P}$ can be represented by an attribute. Each coordinate of this set can be valued from low to high. As an example, $p_i$ can assume values between $[0, 1]$.

Thus emerges the importance of the use and application of Decision Theory in complex projects.

The following will be seen the mathematical concepts of the application of Game Theory to economics.

## 2.3 Game Theory Applications in Economics

A major milestone in the launch of Game Theory was established by von Neumann and Morgenstern, in their book *Theory of Games and Economic Behavior*, which launched the mathematical foundation of this theory for applications in economics, with a background in strategic decisions based on how players act rationally in a common environment, to maximize their real goals through some process of maximization.

The use of the utility function by von Neumann & Morgenstern (1947) to represent the behavior of individuals in pursuit of their goals was stressed for the establishment of the theory. With the assumption of the existence of ordinal utility, the sequential of preference and relationship is key, since the cardinal utility preserves the order relation and also takes into account the intensity of these relationships.

In general, in the Theory of Games a game is defined by a set of rules with at least five components, namely:

i) the number of participants;

ii) the possible actions and strategies;

iii) the results of each player or decision maker;

iv) the function that allows each party to combine his strategies;

v) the relation of preference on each of the results;

In this context, the term game is a process of strategic interaction and maps a conflict situation where the decision-maker, or the player, must make a choice knowing that the result of the player's conflict will be determined in some way by the choices made. The results of these choices will have consequences for decision-makers that can be measured or evaluated as gain, loss or indifference. The risks inherent in each decision may also be estimated.

## 2.3.1 Extensive Games with Perfect Information

Strategic form games have no temporal component. In this case, they are considered an extensive form games with perfect information where each player knows the choice of the others. This is why using a more detailed model in the form of a tree, also called an extensive form game in which players, after some interactions, may over time gain information about the actions of the other players. In an extensive form game with perfect information, each player is at any point aware of the previous choices of all other players. Moreover, only one player moves at a time, so there are no simultaneous movements.

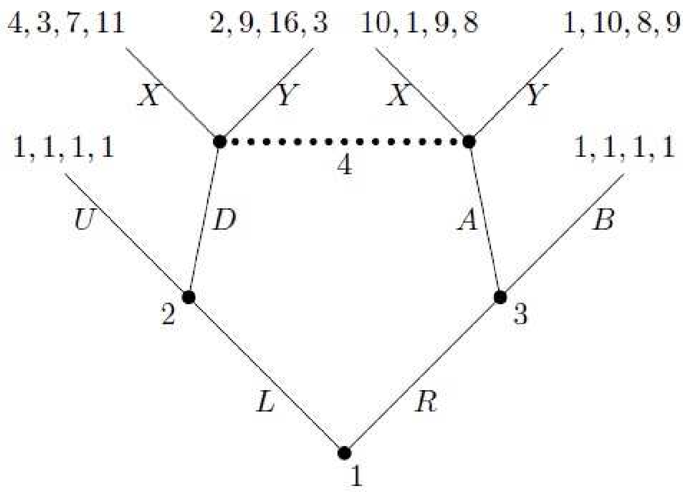


*Figure 2.8: An Extensive Form Game.*

Each point where a player gets to move in the game or at which the game ends is called a node. Nodes at which players move are shown by small black dots in Figure 2.8 and are called decision nodes. The game starts at a particular node, called the initial node or root. In this case we assume that the lowest node where Player 1 moves is the initial node. Player 1 chooses between L or R. If Player 1 chooses L then Player 2 moves and chooses between U or D. If Player 1 chooses R then Player 3 moves and chooses between A and B. If Player 2 chooses U then the game ends. If Player 2 chooses D then player 4 moves. If player 3 chooses B then the game ends. If player 3 chooses A then Player 4 moves. When it's Player 4's turn to move, he doesn't see whether he is called upon to move because Player 1 chose L and Player 2 chose D or because Player 1 chose R and Player 3 chose A. We say that the two nodes at which player 4 moves are in the same information set and we represent this by joining them with a dotted line as in Figure 2.8. If it's player 4's turn to move, he chooses between X and Y, after which the game ends. The nodes at which the game ends are called terminal nodes. To each terminal node we associate a payoff for each player. These payoffs tell us how the player evaluates the game ending at that particular node; that is, they tell us the players' preferences over the terminal nodes, as well as their preferences over randomization of those nodes.

### 2.3.2 Extensive Games with Imperfect Information

Normally, players do not always have full access to all information that is relevant to their decisions. Extensive games with imperfect information model exactly what information is available to players when they make a move. The modeling and evaluation of strategic information is exactly one of the strengths of Game Theory. John Harsanyi's pioneering work in this area was recognized in the 1994 Nobel Awards.

### 2.3.3 The Theory of Principal-Agent

Principal-agent theory was originally applied to the context of private companies to exploit the economic relationship between a manager (the principal) and an employee (an agent). Despite its primary use in the private sector, agency theory can also be applied to the context of the interfaces between the public and the private sector when the public sector (the principal) employs the private sector (the agent) to delegate some of their roles in providing public services. To explain the concept of principal-agent theory, we begin with its definition.

The basic model theory presupposes the existence of two actors or players, known as Principal and Agent, and governance mechanisms and external controls, mediated by a system of compensation, which may be a contract. The components of this model also take into account the incentives, constraints, moral hazard and adverse selection.

For purposes of this work, the principal is the entity responsible for government or the institution responsible for portfolio, that can interact effectively in a larger economic environment, called the market. The approach to principal agent theory applies to project management because of the concept established in the contracting services in the different phases of the project, as shown in Figure 2.9. The strategies in the management of the project are to achieve the goals and objectives of the project. Similarly, the agent also has its own set of actions and payoffs, besides his utility function.

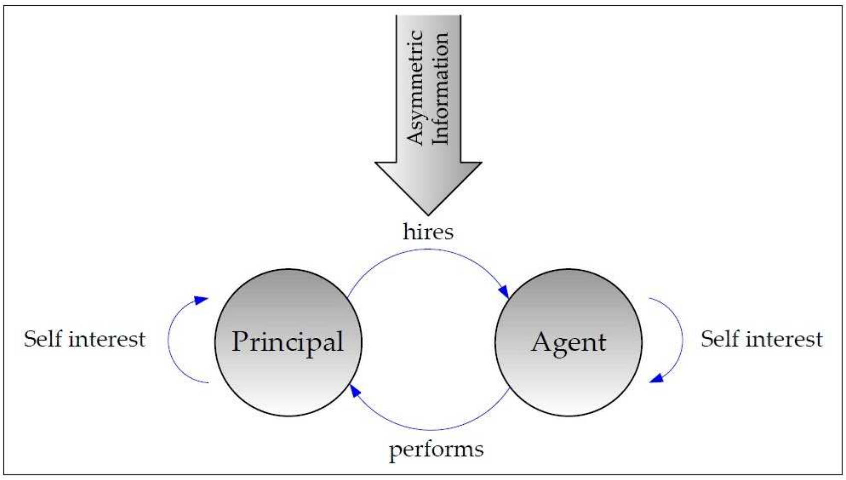


*Figure 2.9: Principal-Agent Theory (Mu, 2008).*

The principal agent theory is a kind of relationship that "is a contract in which one or more persons (the principal) engage another person (the agent) to take actions on behalf of the principal that involve the delegation of some decision making authority to the agent" (Jensen, 2003).

The basic assumption for agency theory is the asymmetric information between the principal and the agent, which induces adverse selection during *ex ante* contracting and moral hazard during *ex post* contracting period.

Adverse selection refers to the fact that the principal is unable to access relevant information about the agent before signing the contract. The moral hazard refers to the fact that the efforts made by the agent dedicated to the task can not be freely observable by the principal and thus causes monitoring problems. The information asymmetry is not necessarily a problem if the agent's interests were perfectly aligned with the principal's. However, the "asymmetry" has actually affected the level of benefits streaming toward the principal. That is because another assumption, goal conflict, exists between the principal and the agent. So when the agent's behaviors are not controlled or restrained, the goals of the principal are unlikely to be attained.

The general formulation of the principal agent theory according to Kreps (1990) in the chapter entitled: "Corporate Culture and Economic Theory", consider:

i) A set of actions of the Principal, $\mathcal{A}_{\mathcal{P}} = \{a_P(1), a_P(2), \ldots, a_P(i), \ldots, a_P(m)\}$, where $i = 1, 2, \ldots, m$. And of Agent, $\mathcal{A}_{\mathcal{A}} = \{a_A(1), a_A(2), \ldots, a_A(j), \ldots, a_A(n)\}$, where $j = 1, 2, \ldots, n$

ii) A set of possible outcomes or payoffs of Principal,

$\mathcal{P}_{\mathcal{P}} = \{p_P(1), p_P(2), \ldots, p_P(i), \ldots, p_P(l)\}$, where $i = 1, 2, \ldots, l$.

And of Agent $\mathcal{P}_{\mathcal{A}} = \{p_A(1), p_A(2), \ldots, p_A(j), \ldots, p_P(r)\}$, where $j = 1, 2, \ldots, r$.

iii) The principal utility function $u_P(w, a_P)$, $w$ being the value of the contract and $a_P$ the action of the principal. Another, the agent's utility function $u_A(w, a_A)$, $w$ as the value of the contract and $a_A$ the action of the agent.

Of course, there are restrictions that can be viewed with the project risks associated with the Principal:

i) The first constraint represents the agent's decision to accept or not the proposed contract by the Principal in terms of other job opportunities available in the market, which can be understood as a useful reserve. The useful reserve leads to a Participation Restriction (PR) or the Individual Rationality constraint.

ii) The second constraint related to the Principal proposes certain additional benefits as a way to encourage producing the desired results, a strategy which is called Incentive Compatibility Rate (ICR) according to Varian (1999).

In both constraints, the project management becomes subject to the risks inherent in their conduct which can result in decisions that increase project costs or have other harmful consequences for the objectives initially outlined, requiring an assessment with respect to the risks involved in these tradoffs.

The objective of the Principal is to make decisions that maximize their utility function, which will correspond to minimizing the risk by choosing an action $a = d(x)$ subject to the restrictions imposed by the optimizing behavior as well as by the agent. When this has a probability distribution on the results $P(p|\theta, d)$, ie, the function result, we have to minimize the risk:

$$\min[R_d] = \min[-u(P(p|\theta, d)] = \min\left[\sum_{\Theta} \pi(\theta)v(p)P(p|\theta, d)\right] \tag{2.3.1}$$

### 2.3.4 Project Management and Strategic and Cooperative Games

In the administration and management of larger projects you can use the concepts and mathematical formulations of Cooperative Games, by the fact that these can be modeled from real situations similar to what happens in both games and in project management. There are situations of conflict, uncertainty, doubts, risks and payoffs to win. Besides that, the strategic framework as a cooperative game between the principal and the agent permit a prior notice before deciding which strategy will be adopted during the game. However, although at the first glance communication seems to facilitate the conduct of contracts, it also opens spaces for coalitions, bluffs and threats that disrupt the production of best results for either part according to Osborne (2004). A key feature is that decisions can be taken at various stages along the project execution so that each player may reconsider its set of actions at each stage of the game at the moment of decision, creating an extensive game, where the set of actions and payoffs are dynamic at each stage of the project. Another feature of large projects is that they lead to a game of imperfect information, because the principal and the agent in reaching a decision, both may have partial information of the actions that have been taken earlier. Because of the great complexity of a project, most likely a player may be uncertain about what action other players have taken during the course of project implementation.

The decision must be made by players at every moment, which is a characteristic of extensive games with imperfect information.

One way to deal with the lack of information in the modeling of the games is to introduce nature as an active player in any move. Thus, the uncertainties of the other players about the definition of the rules can be interpreted as subjective probabilities; that is, the psychology of the players is to establish that according to Harsanyi (1967), systematized this situation by treating agents as "Bayesian" players that is, those whose uncertainties can be operated through a joint subjective probability distribution shared by all.

### 2.3.5 Strategic Games

In game theory, the fact that the players are rational is taken into consideration. A player is considered reasonable based on the way he makes decisions. Aware of its possible actions, the player generates expectations about the unknowns of the problem, has clear preferences and makes the decision after an optimization process. Subject to a scenario, the decision-maker will make a decision based on a greater benefit.

A strategic game consists of:

i) a finite set of M players;

ii) for each player $i \in M$, a set of possible actions not empty $\mathcal{A}_i$;

iii) for each player $i \in M$, there is a utility function that expresses the relation of preference on the choices to be made.

Thus, a strategic game can be seen as a model of an event that occurs only once. Each player knows the details of the game and the fact that other players are rational. Decisions are made independently and simultaneously. From this point of view, a player is not aware of the decisions of others in reaching its decision. At most, expectations can be formed based on general information about the game.

**Definition 2.3.1** *: Game Strategy,* $\Gamma = (G, \mathcal{A}_i, u_i)$*, (Myerson, 1997), where:*

i) $G$ is a finite set of players;

ii) $\mathcal{A}_i$ represents the set of player's actions, $i \in G$;

iii) $u_i$ represents the utility function of each player $i$.

Each player $g_i \in G$ has a finite set $S_i = \{s_{i1}, s_{i2}, \ldots, s_{ir}\}$ of options, called pure strategies of player $g_i(m_i \geq 2)$, where $r = m^t$, as per in equation 2.2.6. A vector $s_i =$

$[(s_1), (s_2), \ldots, (s_M)]$ and $(s_i)$ is a pure strategy for player $g_i \in G$ is called a pure strategy profile, where M is the number of players.

The set of all pure strategy profiles form therefore the cartesian product:

$S = \prod_{i=1}^{N} S_i = S_1 \times S_2 \times \cdots \times S_N$, called the space of pure strategy game. $N$ is the number of decision stages.

For the player $g_i \in G$, there is a utility function.

$$u_i : \underset{s \longmapsto u_i(s)}{S \to \mathbb{R}} \tag{2.3.2}$$

which combines the gain (payoff) $u_i(s)$ Player $g_i$ to each pure strategy profile $s \in S$ according to Vasconcelos (2007).

In the case of principal agent theory, the game has two players, a principal and an agent, represented by $G = \{g_P, g_A\}$.

**Definition 2.3.2** *: A Strategic Extensive Game with Perfect Information is defined by $(G, H(a_i), u_i)$, where:*

i) $G$ is a finite set of players;

ii) $H(a_i)$ represents the set of actions called the sequential history of actions;

iii) $u_i$ represents the utility function of each player $i$.

A strategic solution or Nash Equilibrium of a game is a point where each player has no incentive to change his strategy if the other players do not.

**Definition 2.3.3** *: A strategy profile $s^* = (s_1^*, \ldots s_{(i-1)}^*, s_i^*, s_{(i+1)}^*, \ldots s_r^*) \in S$ is a Nash Equilibrium if, for each i, his choice $s_i^*$ is the best response to the other players choices $s_i^*$, thus:*

$u_i(s_i^*, s_{-i}^*) \geq u_i(s_{ij_i}^*, s_{-i}^*), \forall i = 1, \ldots r$ and $\forall j_i = 1, \ldots, m_i$, with $m_i \geq 2$

# 3 THE LOCI AND THE GENESIS OF UNCERTAINTY

Uncertainty has accompanied mankind since the days of life in caves. While uncertainty is present in a variety of current situations, is not what restricts mankind's actions with relationship to other humans. There are other conditions that influence human relationships: such as risks, actions, knowledge and other attributes. But you can say that uncertainty has a great influence on decisions and human relationships. Man seeks to understand the world as a system. Man looks at each case as connection to the past that can give answers to questions about the future (Lieber, 2003).

Human action as a result brings a new start, where uncertainties are renewed, and new actions will be taken, creating a continuous process, where the field of action in the present moment is always targeting the future.

The economist John Maynard Keynes, describes uncertainty as:

> By uncertain knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty . . . Or . . . the expectation of life is only slightly uncertain. Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence . . . About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know, (Keynes, 1937).

In this sense, Keynes poses phenomena as uncertain, those things for which there is no scientific basis for assigning probabilities. In turn, Lawson (1988) presented a taxonomy dividing uncertainty as likely measurable and not measurable. More recently, Liu (2011) presented a mathematical construct for the theory of uncertainty that may be seen in Figure 3.1. Three basic concepts are used to construct the theory of uncertainty: a measure of uncertainty to indicate the degree of belief that an uncertain event can occur, an uncertainty that assumes variable values to represent amounts of uncertainty and the distribution of uncertainty are used to describe the behavior of the uncertainty.

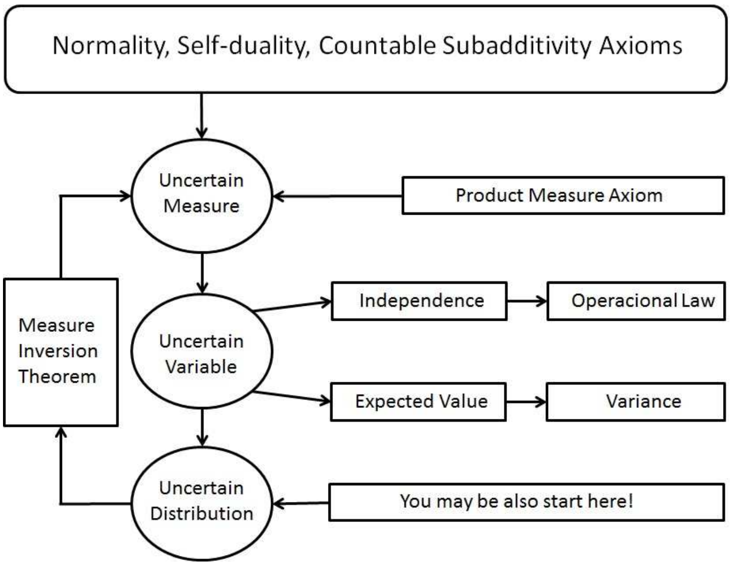


*Figure 3.1: Uncertainty Theory (Liu, 2011).*

## 3.1 A Measure of Uncertainty

Let $\Gamma$ a non-empty set. A collection $\mathcal{L}$ of subsets $\Gamma$ is said to be a $\sigma - algebra$ if:

i) $\Gamma \in \mathcal{L}$.

ii) If $\Lambda \in \mathcal{L}$, then $\Lambda^c \in \mathcal{L}$, and

iii) If $\Gamma_i, i = 1, 2, ... \in \mathcal{L}$, then $\Gamma_1 \cup \Gamma_2 \cup ... \in \mathcal{L}$

Thus, each element $\Gamma_i$ is a $\sigma - algebra, \mathcal{L}$ is called an event, and to ensure that $\mathcal{M}\{\Lambda\}$ is a measure of uncertainty for this event $\mathcal{L}$, in the interval [0,1], Liu (2011) proposed the following axioms:

**Axiom 3.1.1** ***Normality***: $\mathcal{M}\{\Gamma\} = 1$, *for the whole set* $\Gamma$

**Axiom 3.1.2** ***Self-Duality***: $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$, *for every event* $\Lambda$

**Axiom 3.1.3** ***Countable Subadditivity***:

$$\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\} \tag{3.1.1}$$

A measure of uncertainty is interpreted as the degree of personal belief in an event that may occur. The self-duality ensures that the theory of uncertainty is consistent with the law of the excluded third and the law of contradiction.

Although the probability measure satisfies the above three axioms of probability theory it is not a special case of the theory of uncertainty.

**Axiom 3.1.4** ***Product Measure***: *Let* $(\Lambda_k, \mathcal{L}_k, \mathcal{M}_k)$ *be the space of uncertainty for* $k = 2, 3, .., n$. *Then the product of the uncertain measure* $\mathcal{M}$ *is a measure of uncertainty about the product of* $\sigma - algebra$ *with* $\mathcal{L}_1 \times \mathcal{L}_2 \times \cdots \times \mathcal{L}_k$*, satisfying:*

$$\mathcal{M}\{\prod_{k=1}^{\infty} \Lambda_i\} = \min \mathcal{M}_{1 \leq k \leq n}\{\Lambda_k\} \tag{3.1.2}$$

## 3.2 Uncertainty Variable

The concept of the uncertainty variable in this model is to represent uncertain phenomena that can be relatively invariant in relation to the range of human estimates.

**Definition 3.2.1** *: A variable of uncertainty is a measurable function* $\xi$ *of an uncertain space* $(\Lambda_k, \mathcal{L}_k, \mathcal{M}_k)$ *for a set of real numbers, i.e., for any Borel set* $\mathcal{B}$*, the set:*

$$\{\xi \in \mathcal{B}\} = \upsilon \in \Gamma | \xi(\upsilon) \in \mathcal{B} \tag{3.2.1}$$

is an event.

The variable of uncertainty is different from the definition of the random variable defined by Kolmogorov and fuzzy variables defined by Zadeh (1965).

**Definition 3.2.2** *: A vector n-dimensional is a measurable function from the space of uncertainty* $(\Lambda_k, \mathcal{L}_k, \mathcal{M}_k)$ *in the set of n-dimensional real vectors, i.e., for any Borel set* $\mathcal{B} < n$ *of* $\mathbb{R}^n$ *the set:*

$$\{\xi \in \mathcal{B}\} = \upsilon \in \Gamma | \xi(\upsilon) \in \mathcal{B} \tag{3.2.2}$$

is an event.

**Theorem 3.1** *The vector* $(\xi_1, \xi_2, ..., \xi_n)$ *is a vector of uncertainties if and only if* $(\xi_1, \xi_2, ..., \xi_n)$ *variables are uncertain.*

## 3.3 Uncertainty Distribution

Uncertainty distribution describes the chance that a variable can take over a space of values.

**Definition 3.3.1** *: The distribution of the uncertainty* $\Phi$ *of a variable* $\xi$ *is defined by:*

$$\Phi_\xi(x) = \mathcal{M}\{\xi \leq x\} \tag{3.3.1}$$

for any real number $x$.

**Theorem 3.2** *The function* $\Phi : \mathbb{R} \longmapsto [0, 1]$ *is an uncertainty distribution, if and only if, it is a monotonic increasing function, except for* $\Phi(x) \equiv 0$ *and* $\Phi(x) \equiv 1$.

For example, the uncertainty distribution for a generic function can be viewed in figure 3.2

## 3.4 Uncertainty in Models of Decision Theory

The constructs of mathematical decision theory show that there are four sets of variables: the payoff $\{\mathcal{P}\}$, the state of nature $\{\Theta\}$, the observations $\{\mathcal{X}\}$ and the set of actions

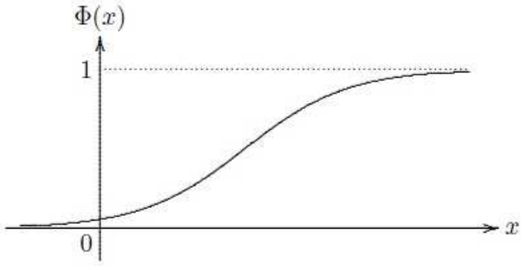


*Figure 3.2: Example Graph of an Uncertainty Distribution.*

$\{\mathcal{A}\}$, that influences the decision for each scenario. What follows are mathematical inserts to be considered for uncertainty in each of the four sets.

For all the payoffs, each element of the set $\{\mathcal{P}\}$, would be comprised of:

$\mathcal{P}$ $=\{p_1, p_2, \ldots, p_i, \ldots, p_l\}$, where the i-th term of the set is $p_i = p_{i(\text{true})} + \xi_i$, $i = 1, 2, \ldots, l$, and $p_i$ would be given by an real payoff amount $p_{i(\text{true})}$ plus an amount $\xi_l$ representing the uncertainty over this element, where $\xi_i$ follows a probability distribution $\Phi_{\mathcal{P}}(\xi)$.

Similarly, the set of states of nature $\{\Theta\}$, after the inclusion of uncertainty:

$\Theta = \{\theta_1, \theta_2, \ldots, \theta_i, \ldots, \theta_n\}$, where the i-th term of the set is $\theta_i = \theta_{i(\text{true})} + \xi_i$, $i = 1, 2, \ldots, n$, would be given by an amount representing the real state of nature $\theta_i = \theta_{i(\text{true})}$ plus an amount $\xi_i$ representing the uncertainty about this element, where $\xi_i$ follows a probability distribution $\Phi_{\Theta}(\xi)$.

For all the observations $\{\mathcal{X}\}$, the inclusion of uncertainty results in:

$\mathcal{X} = \{x_1, x_2, \ldots, x_i, \ldots, x_t\}$, where the i-th term of the set is $x_i = x_{i(\text{true})} + \xi_i$, $i = 1, 2, \ldots, t$, a portion would be given by real observation $x_{i(\text{true})}$ plus an amount $\xi_i$ representing the uncertainty over this element, where $\xi_i$ follows a probability distribution $\Phi_{\mathcal{X}}(\xi)$.

Finally, the set of actions $\{\mathcal{A}\}$ with the inclusion of the variable of uncertainty, would be:

$\mathcal{A} = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$, where the i-th of the set, $a_i = a_{i(\text{true})} + \xi_i$, $i = 1, 2, \ldots m$, a portion would be given by real observation $a_{i(\text{true})}$ plus an amount $\xi_i$ representing the uncertainty over this element, where $\xi_i$ follows a distribution probability $\Phi_{\mathcal{A}}(\xi)$.

The result of the probabilistic mechanisms problem are the consequence function, the likelihood function and the prior distribution of states of nature. These functions have uncertainty incorporated in their models of probability distributions, thereby mathematically guaranteeing the inclusion of the variable of uncertainty in models of Decision Theory.

# 4 THE PROBLEM

A project can be defined as a set of coordinated activities, usually sequential and interrelated, aiming to create a product, service or a unique result, at a given time with a specific life cycle, with certain resources and defined responsibility.

For project management, it is understood that the use and application of technical knowledge and skills ensures that a project is successful, passing through the stages of planning, execution and control activities.

In the context of project management, the concept of problem can be stated as a difference between actual production (results produced) and the anticipated production (planned results) or possible production (a result that can be obtained using optimization processes).

The analysis of a problem situation can be identified by means of diagnosis to find the cause and effect or any method to identify such situations, for example as with the Balanced Scorecard (BSC). This was a concept developed by Kaplan and Norton in the 90's that can be understood as a management tool, composed of a strategy map, strategic objectives, indicators, targets and initiatives.

According to the PMI (2004) [1], North American Institute that issues recommendations on best practices for project management, there are nine specific areas of knowledge that can manage and track large projects , as follows:

1. Integration Management;
2. Scope Management;
3. Time Management;
4. Cost Management;
5. Quality Management;
6. Human Resource Management;
7. Management Communications;

[1] Project Management Institute

8. Risk Management;

9. Acquisition Management.

However, in most projects, the attributes concentrate on the following: Scope, Time, Cost and Quality (Turner, 1993).

A complex project usually has a large size, with high investment values, a long execution time and quality requirements. This requires the manager to control variables, attributes and features to achieve specific goals.

To the extent that the number of observations grows and the number of actions also increase, the decision options follow the relationship in equation 2.2.6, or:

$$||\mathcal{D}|| = ||\mathcal{A}||^{||\mathcal{X}||} \tag{4.0.1}$$

This makes it very difficult to make a decision, thus requiring a risk management assessment for the project.

For example, risk assessment is a vital process in any effective information system development. In fact, risks are intrinsic to any project and risk-taking is a necessary component of any process of decision making. Poor risk management of information systems on a project often leads to failure, a situation not uncommon in both the public and corporate community (Zhou, 2008).

Thus emerges the concept that in order for the decision-maker to achieve the best goals, he will have to make choices. He can earn more in some aspects or attributes and lose in others, resulting in compensation aspects or attributes to be worked out by the project manager. The difficulties that present themselves in managing these projects is the high degree of uncertainty surrounding all the attributes, with inclusion of uncertainty in organizational areas, technical areas and processes according to Leijten (2010). On many occasions, the project manager is confronted with situations where he could get a good result at a high risk. Thus, it can be said that manager is an individual who has a propensity for taking-risks. Otherwise, the manager could get a poor payoff return with a lower risk, and in this case the manager has the characteristics of risk aversion. This work aims at studying and implementing the management decisions on complex project with uncertainty associated with risks.

One of the biggest problems of conducting large projects mentioned in the literature is with communication management, considered one of the causes of the problems encountered in project implementation of RandstaRail (Leijten, 2010).

Under the mathematical framework of Decision Theory and Game Theory, a project management policy will be proposed, with emphasis on joint strategies, based on the calculation of the risks associated with management decisions, taking Bayes Risk as a measure of the cumulative values.

## 4.1 Environments Influencing the Generation of Uncertainties Associated with Risk in Complex Projects

The recent history many situations have been seen in which global institutions have lost billions of dollars, because of the lack of effective control of the risks involved in their investments. Companies such as Long Term Capital Management (LTCM), Barings, Procter & Gamble, Orange County and Daiwa are some of the classic examples. Add to these episodes, the recent crises undergone by the international financial market and the increasing development of derivatives, the management and control of financial risks becomes a concern for investors, financial institutions, businesses and regulators. The importance of risk management has been growing steadily since the mid-90 (LaRocque, 2003) and may have an influence on emerging risks in complex projects.

Another situation occurred in Brazil, involving several companies that make decisions for the implementation of their projects. They have made decisions which led to very costly errors, according to an article in the magazine “Exame” in 2005, entitled: The cost of a wrong decision (Albuquerque, 2011). The following table shows the companies, the decisions, the errors and the costs involved:

*Table 4.1: The Cost of a Wrong Decision (Exame, 2005 apud Albuquerque, 2011).*

| COMPANY | DECISION | MISTAKE | COST |
|---|---|---|---|
| Mercedes-Benz | Installing a factory for the production of a Class A car in Juiz de Fora, MB, Brazil | Wrongly sizing the market and brought a automobile that did not please Brazilians | $ 500 million (Estimated loss |
| General Motors | Signing an agreement with American auto-workers to ensure stable employment and health insurance and offered private pensions that were too generous | Not realize that long-term agreement limits the company and causes a high financial loss | $ 5.6 billion (Annual expenditures with health insurance and pensions) |
| Merck | Selling the anti-inflammatory Vioxx knowing that it could cause cardiovascular disorders | Belittled the problem. Just pulled Vioxx off the shelves when there was no alternative after one year. | $ 28 billion (Fall in market value of Merck between August 2004 and end of 2005). |
| Hewlett-Packard | Acquisition of rival Compaq | The merger of the companies did not work. Sales have doubled but profits remained unchanged. | $ 19 billion (Amount of purchase). |
| IBM | Buying a Microsoft operating system to equip their personal computers. | IBM did not foresee they would become hostage to Microsoft | $ 75 billion (Loss of market value of IBM between the 80's and 90's). |

In a macro view, there are many variables that can influence the decision-maker when facing complex projects, namely:

i) Type of contract between the Principal and Agent.

ii) Contract Types and Financial Risk Allocation

iii) Risks Inherent in Project Management.

Below is specific information on each of these subjects.

### 4.1.1 Types of Contract between the Principal and Agent

An important aspect of project management which should always be considered, is the type of contract established between the principal and the agent. It involves the Public Administration and Private Enterprise as in the cases studies which served as motivation for this work in Leijten (2010) and Palma (2009). In public administration, one can enumerate some types of contracts, namely:

A) **Administrative Contract**

Private contracts are those that the Government is partnering with the private sector, stripping itself of the power of eminent domain, and placing the private sector on an equal footing in the contract. Such contracts do not have the final objective of achieving the goals as a major duty of the state. Examples of such contracts are leases for the use as offices and the purchase and sale of materials. Private contracts are of course governed by civil or commercial rules. In Brazil, these contracts are governed by law number 8.666/93.

Public-Private Partnerships (PPP) contracts are this kind of a special type of contract implemented by Brazilian Ministry of Planning according to Ministério do Planejamento (2011). The PPP is a contract to provide services over the medium and long term (5 to 35 years). The values of these contracts are values above $12 million. It is prohibited to include single objectives, the supply of manpower, equipment or execution of public works in these contracts.

In PPP, the deployment of infrastructure necessary for the service contracted by the government will depend on funding initiatives from the private sector and individual remuneration will be determined based on performance standards and will be due only when the service is available to the State or users. Brazilian law provides the possibility of combining the remuneration tariff with the payment of public healthcare plan and defines PPP as an administrative licensing contract with sponsored or administrative procedures.

B) **Covenants**

The Covenant is a legal instrument that governs the transfer of public resources. Has the Federal Government participates with the state governments, foundations, public companies or mixed agencies. The financial resources are from the treasure of the Union, to implement work programs, projects or events of mutual interest in mutual cooperation (STN Instruction 001/97).

C) **Transfer Agreement**

The Transfer Agreement is a European Instrument for the transfer of funds to states, municipalities and the federal district, by Decree number 1.819/96.

D) **Term Partnership**

The Term Partnership is is a new legal instrument created by law number 9.790/99 (art. 9) to perform a unique partnership between the Government and OSCIP for the promotion and implementation of projects. In other words, a Partnership consolidates a cooperation agreement between the parties and constitutes an alternative to an agreement for the realization of projects and OSCIPs agencies among Civil Society Organization of Public Interest at the three spheres of government, featuring simpler procedures than those used for the conclusion of an agreement.

### 4.1.2 Contract Types and Financial Risk Allocation

In the professional and business sense in the world generally, the a contract is an instrument that formalizes the business and interests of parties involved. Despite the evolution over time of these formalizations, the fundamental basis of these documents remains the same, ie, the allocation of risks. Several provisions are included in order to allocate and manage varieties and types of risks. Usually when a clause allocates an certain amount of risk on to one party, the other party is not exposed in the same level of risk. The following Figure 4.1 shows how this sum of risks can have a balance to can be balanced between the contracting party and the contractor, depending on the type of contract. Thus, the choice of contract and its clauses can have an influence on emerging risks in complex projects.

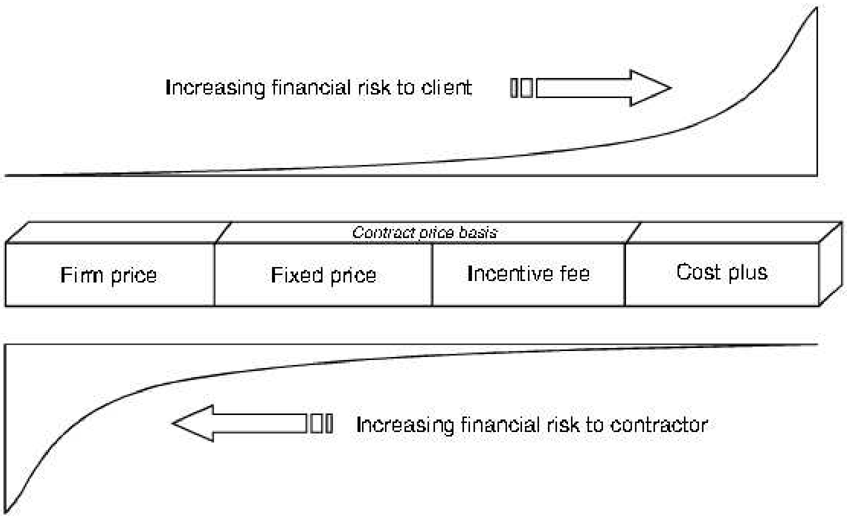


*Figure 4.1: Contract Types and Financial Risk Allocation (Cooper, 2005).*

Over time, the business world has developed agreements with contracts involving many parts of complex documents. Today's executives are able to choose between a large number of types of contracts that have been developed by experience to allocate and manage a variety of risk under a variety of circumstances. When considering a particular type of contract, it can be assumed that when a contract allocates a great deal of risk for to one party, the other party is not normally exposed to the same level in this area.

## 4.2 Risks Inherent in Project Management

Risk Management is an activity that is increasingly taking up space in organizations that manage complex projects. The objective is the prevention of accidents and minimizing their consequences. After studying the identification and treatment of risks and developing a plan of action, a solution is sought for possible harmful effects to the project.

The exact timing for initiating treatment of risk by means of an action plan, is contextualized in Figure 4.2:

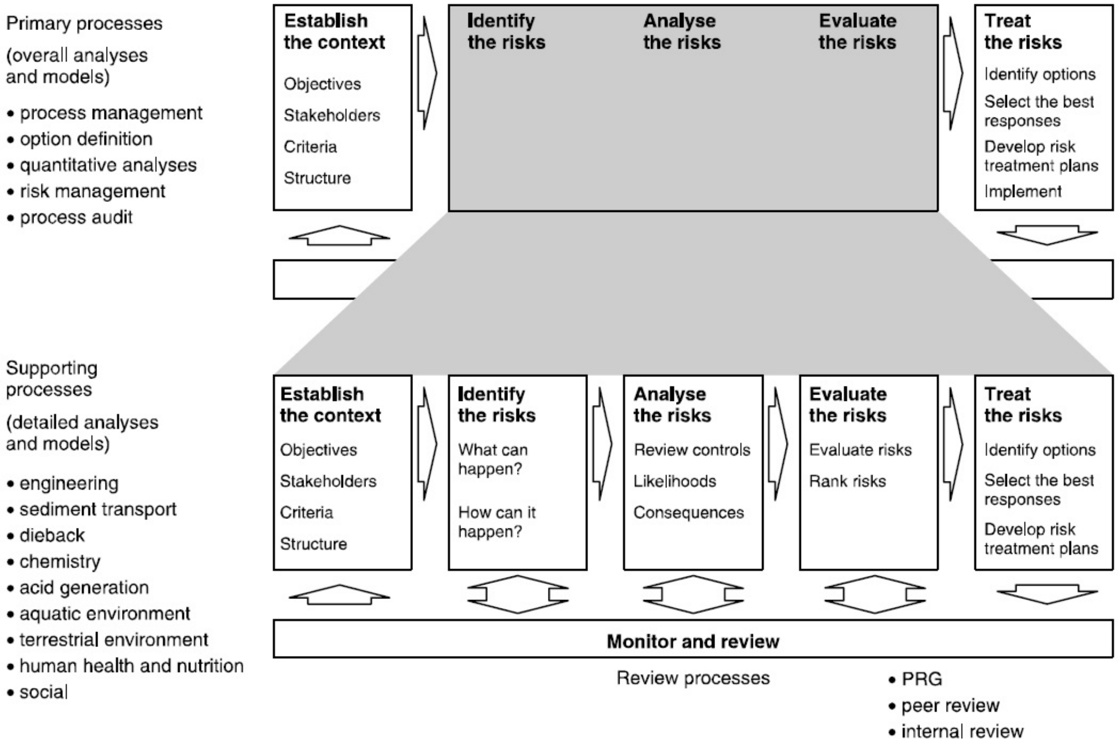


*Figure 4.2: Context of Risk Management in Projects (Cooper, 2005).*

The risk management process incorporates full knowledge of the project which is to be executed, its goals, stakeholders, the criteria to be adopted in project management and operating structure. Only after knowing this information, the operation of risk management itself can begin, with the identification, analysis and estimation of the risks involved in the project.

The source of risks can be better explained through Risk Analytical Structure, which is specific to each project and each structure, shown as an example in Figure 4.3:

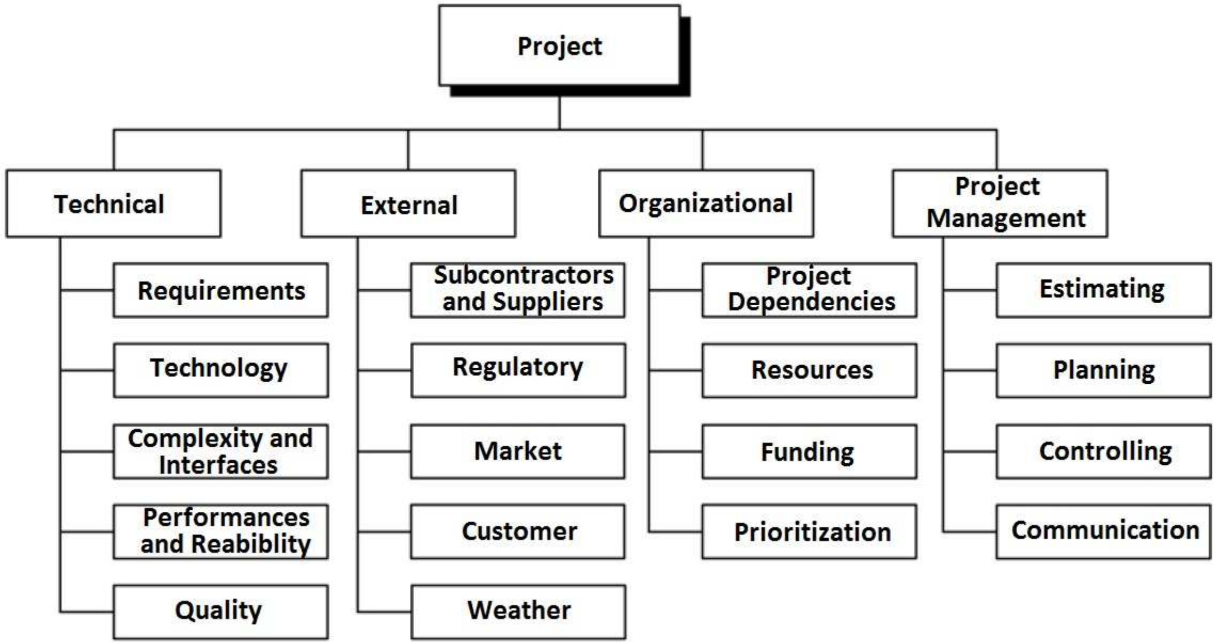


*Figure 4.3: Example of a Risk Analytical Structure for a Project (PMI, 2000).*

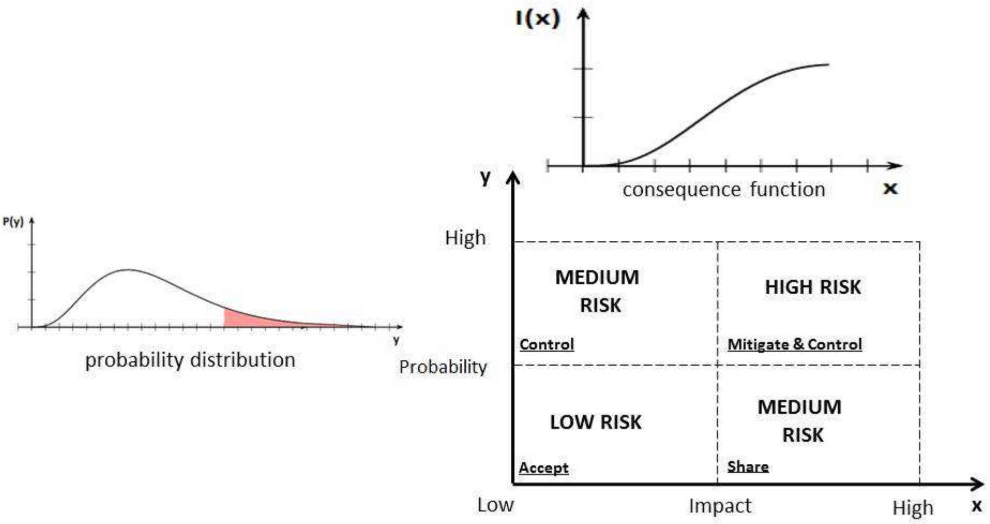


*Figure 4.4: Vulnerability Matrix (Cooper, 2005, adapted).*

The external source of risk relates to the nature of states Θ, defined above in the theoretical framework.

In the analysis stage, the process is concerned with classification and risk estimate, frequency of occurrence, intensity, significance and consequence. Thus, the process may consider historical data, empirical data, theoretical analysis and analysis of by experts. In which case the an estimate of the a prior probability distribution $\pi(\theta)$ on the states of nature can be considered.In the phase of identifying risks, it looks for the source of risk, nature and impacts that might affect the project. Existing techniques, methodologies and tools for such identification, which may, through the experiences (data) of other similar projects analyze all parts of a process to identify the risks.

The qualitative analysis takes into account the nominal or descriptive scales to describe the probability and consequences of the risks, while the quantitative analysis uses numerical ratio scales of probabilities and consequences, rather than descriptive scales. The goal is to find a classification for the risk as low, medium or high according to the probability of occurrence and relative impact, as illustrated in figure 4.4:

The risk is mathematically related to the concepts of probability and impact, of a potential event or situation that may cause negative or positive impacts on the project, depending on the decisions taken. Risk is an objective measure of uncertainty that can be defined as a function of two basic components.

$$\text{Risk} = \text{f(impact, probability)}$$

where:

i) The probability of uncertainty: Is the probability of failure to produce a specific result or occurrence of an undesired event.

ii) The impact of uncertainty: the impact on the expected outcome in the event of a failure to produce a specific result or occurrence of an undesired event.

Although each risk is related to uncertainty, uncertainty can not alone represent a risk.

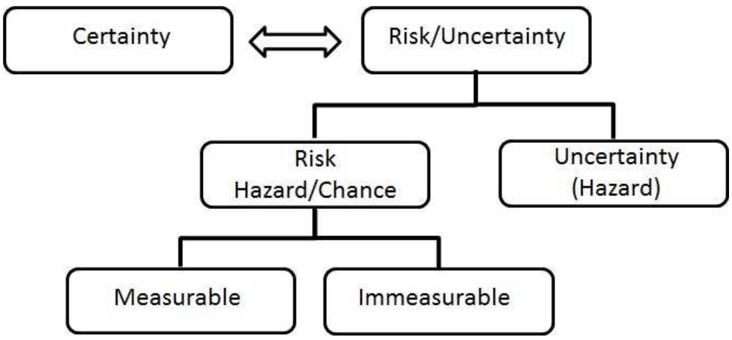


*Figure 4.5: Limits of Risk and Certainty (Demir, 2010).*

Here the distinction is the level of interaction between uncertainty and the process required to meet the objective, as observed in Figure 4.5.

Therefore, the future returns of a project subject to complex risks can only be estimated through the probability distribution, because a probability distribution reflects the variability of possible future returns.

Risk analysis is a methodology that assumes the existence of factors that determine the profitability of a project. There are future events that can not be predicted accurately.

The purpose of the identification of risks (or its variability) is to give greater driving safety and protection to the project investments. After this identification, the risks can be analyzed, classified and treated by means of action plans.

Risk assessment and analysis are closely related, so most of the techniques used to measure risk are also used for risk analysis according to Demir (2010).

The risk may be rated as the combination of the probability of an event and its consequences. For each identified risk, there is a probability of occurrence associated with the event, featuring a quantitative analysis of risk, which may be listed in an order of highest to lowest probability. The magnitude of impact suggests a qualitative analysis of risks, according to that shown in Figure 4.4.

Of course, the process of risk management is increasingly recognized because of concern about the positive aspects and negative aspects of risk as well as its intensity. Part of risk management is focused both on preventing the possibility of such events, but also on mitigating the damage after the occurrence of the event.

There are times when the decision maker or manager must make decisions in situations of high complexity. These situations may relate to project management for implementation of benefits to the population, or the deployment of a service

The literature has reported throughout its long history various disasters, some of them generated by natural geological phenomena, such as landslides, volcanic eruptions, which are classified as geological risks, rains and floods, as hydrometeorlogical hazards, asteroid impact, global and geological risks. Many of these phenomena have causes that do not depend directly on man. However, there are a number of other disasters which rely on human participation, and culminate in loss of life, material losses, as well as heavy losses to the environment. Behind these disasters are usually governments and corporations.

The problems presented here could be resolved or minimized through a policy on risk management, based on Decision Theory and Game Theory in Economic Applications. This will be seen in the following chapter.

# 5 THE SOLUTION USING DECISION THEORY

In complex projects, a series of decisions are taken in sequence over time, and, for a project to be successful, every decision must be optimal, minimizing risks in order to have a successful strategy to achieve the objectives and goals of projects. Thus, an optimal strategy is proposed based on the concepts of Decision Theory, known as sequential decisions, according to Campello de Souza (2007b) and the Theory of Games.

## 5.1 Minimum Risk Strategic Policy, Using Decision Theory for Managing Complex Projects

The proposal for a strategic policy of minimal risk for complex projects, assumes that there are several scenarios in the implementation of these projects. In each scenario, decisions must be made. Each of these scenarios has a set of information about the project, represented by the sets: payoffs, states of nature, observations and actions, as shown in Figure 5.1. The decision maker will consider the information contained in each scenario and make decisions, considering the mathematical constructs of the Theory of Decision. However, the optimal decision is one that minimizes the risk and therefore maximizes the utility function.

With these principles will be formulated a policy of minimum risk management applied to complex projects. The approach of the Theory of Games and Economic Applications of Principal-Agent Theory will be considered.

To formulate a strategic policy of minimum risk, the stages of project implementation (order k) will be involved in extensive games with imperfect information. The decisions at each stage resolve or minimize the sources of problems listed in section 4.1.

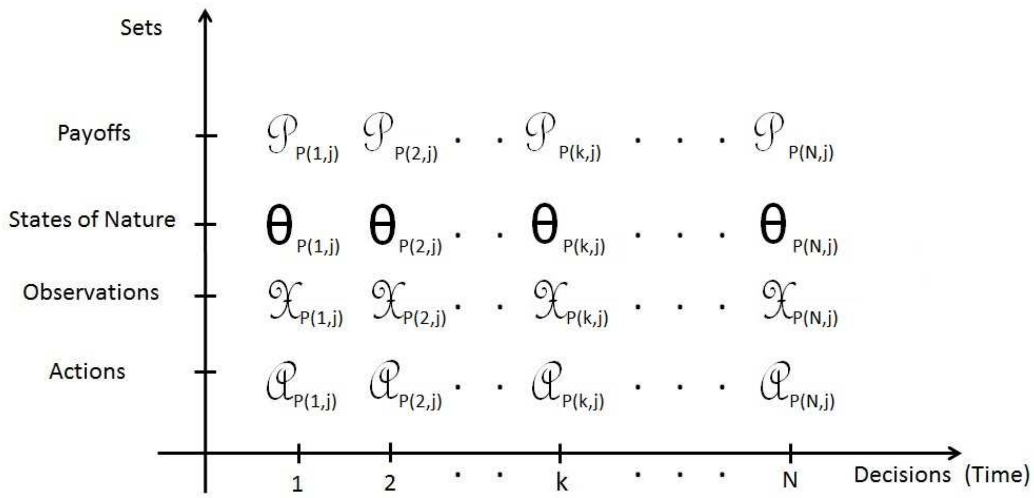


*Figure 5.1: Several Scenarios for the Decision on Complex Projects in k Steps.*

Some considerations can be made regarding the scenarios shown in Figure 5.1, such as:

- the decision maker's utility function is time invariant;
- for each set of a construct of Decision Theory, probability distribution are generated over time;
- in every decision, the data are changed to represent new situations before the decision is finalized.

Considering the various scenarios of Complex Projects a set of steps is proposed to build an optimal strategy for the decision maker. The ideal would be to have all data stored in a database managed by a decision support system.

1. Overview of Principal

   i) States of Nature:

   $\Theta_P$(i,j) $= \{\theta_P(i,1), \theta_P(i,2), \ldots, \theta_P(i,n)\}$, where $i = 1, 2 \ldots, k, \ldots, N$ represents the parameter of the moment of decision in time. The parameter $j$, define the dimension of the vector of states of nature.

ii) Actions Set:

$\mathcal{A}_{\mathcal{P}}$(i,j) $=\{a_P(i,1), a_P(i,2), \ldots, a_P(i,m)\}$, where $i = 1, 2 \ldots, k, \ldots, N$ represents the parameter of the moment of decision in time. The parameter $j$, define the size of the vector of actions.

iii) Goods or *Payoffs* Set:

$\mathcal{P}_{\mathcal{P}}$(i,j) $=\{p_P(i,1), p_P(i,2), \ldots, p_P(i,l)\}$, where $i = 1, 2 \ldots, k, \ldots, N$ represents the parameter of the moment of decision in time. The parameter $j$, define the size of the array of goods.

iv) Observations Set:

$\mathcal{X}_{\mathcal{P}}$(i,j) $=\{x_P(i,1), x_P(i,2), \ldots, x_P(i,t)\}$, where $i = 1, 2 \ldots, k, \ldots, N$ represents the parameter of the moment of decision in time. The parameter $j$, define the dimension of the vector of observations.

2. If the decision maker's utility function is not known, the eduction methods should be implemented to establish the utility function $u(p)$, using one of the methods described in subsection 2.2.4.

3. Probabilistic mechanisms need to be established to determine the consequence function $P(p|\theta, a)$, the likelihood function $P(x|\theta)$ and the prior distribution of states of nature $\pi(\theta)$. These data can be obtained through historical series or through a specialist.

4. With the known decision maker's utility function, the expected utility of distributions on $P$, can be calculated and parameterized by the state of nature $\theta$ and the sets of possible actions $\mathcal{A}$, thus generating all the possibilities of the expected utilities of $u(P(p|\theta, a)$, where $a = d(x)$.

5. For the calculation of the loss function, to support decision-making the main thing is: to choose an actions $a_P$(i,j) $= d_P(x_{i,j})$, as probability distribution of these events, represented by the consequence function, according to the equation 2.2.7, which also takes into account the decision maker's Profile with its utility function, ie:

$$L_P(i,j)(\theta_P(i,j), d(x_P(i,j))) = -u(P(p_P|\theta_P, d_P(x))) \tag{5.1.1}$$

Thus, the action to be chosen should be one that produces the smallest loss, or greatest payoff for the main decision makers (if there is propensity to risk, indifferent or risk averse), thus choosing the loss function is very important for selecting the lowest risk:

$$L_P(i,j)(\theta_P(i,j), d_P^*(x_P(i,j))) = Min[-u(P(p_P|\theta_P, d_P(x)))] \tag{5.1.2}$$

by an optimal choice of $d_P(x)$ and $u(P(p_P|\theta_P))$, called $d_P^*(x)$.

6. Calculating the Bayes Risk: With the loss function which represents a gain or damage in the project due to an action taken by the Decision Maker (principal), calculate the Bayes Risk with the knowledge of the likelihood function $P_P(i,j)(x_P(i,j)|\theta_P(i,j))$, with the expression 2.2.8:

$$r_P(d)(\theta_P(i,j)) = \sum L_P(\theta_P, d_P(x))P_P(x|\theta_P) = E(L|\theta, d) = E_\theta[(L(\theta, d(x))] \tag{5.1.3}$$

The Bayes risk can also be calculated by:

$$r_P(d) = -u(P(p|d)) = \sum_{\theta\in\Theta} \pi(\theta)u(P(p|\theta, d)) = \sum_{\theta\in\Theta} \pi(\theta)R_d(\theta) \tag{5.1.4}$$

The best decision rule is chosen by minimizing $r_d$, and the variable choice of a deterministic decision rule $d$. It is he rule $d$ that minimizes the Bayes risk and is the Bayes rule. Mathematically it is formed as follows:

$$r_P^*(d) = \min_{\{d\}}[r_P(d)] = \sum_{\theta\in\Theta} \pi(\theta)R_d(\theta) \tag{5.1.5}$$

The steps 1 through 6 show the traditional sequence of Decision Theory. Steps 7 and 8 form the optimal policy proposal.

At this point, in step $i$, with the action taken, the strategy is characterized by the player (principal and project manager). The strategy is chosen from the set $S_{ij} = \{s_{i1}, s_{i2} \ldots, s_{ir}\}$, where $r = m^t$, as per equation 2.2.6. Among the options called pure strategies minimal risk of the player, represented by $s^*_{ij}$. Thus, the optimal strategy for the principal is represented by $S^*_{ij} = \{s^*_{ij}\}$. So every step of the project is to form the set of all pure strategy profiles of minimum risk, forming the cartesian product of minimal risk:

$S^*(i,j) = \prod_{i=1}^{N} S^*_{ij} = S^*_{1j} \times S^*_{2j} \times \cdots \times S^*_{Nj}$, where $N$ is the number of decision stages. $S^*(i,j)$ is called the space of pure strategy game, thus defining the minimum risk policy project, part of the principal.

Of course, the optimal strategy takes into account also the Nash Equilibrium due to the fact that the chosen action represents the greatest utility of the principal, hence the lower risk of Bayes, even considering the different profiles of the decision makers (propensity risk, indifferent or averse to risk), ie:

$u_P(i,j)[s^*_{ij}, s^*_{i-j}] \geq u_P(i,j)[s^*_{ij_l}, s^*_{ij_{-l}}], \forall j = 1, \ldots n$ and $\forall l_j = 1, \ldots, m_j$, with $m_j \geq 2$. However, in some games the Nash Equilibrium can not be reached, but the choice does not invalidate the principal action of the lower risk.

7. Calculation of cumulative sums of the Bayes risk. In each scenario, a set of decision rules of dimension $d$ will be generated. The computational effort in each step must be calculated for all combinations of the sums accumulated, providing a set of $d^k$ cumulative sums. In stage $k$, we have:

$$R_{acc}(k) = \sum_{i=1}^{k} r_{P_i}(d). \tag{5.1.6}$$

8. Process sliding window. As the number of stages increases, one can use the concept of the sliding window and consider together the set of accumulated sums of the last stages, thereby reducing the computational effort, where $N - W$ represents the size of the window, $(W < N)$. In this case,

$$R_{acc}(N) = \sum_{i=N-W}^{N} r_{P_i}(d). \tag{5.1.7}$$

9. Calculation of the metric of optimal policy design. The proposal is to calculate the Minimum Cumulative Bayes Risk, when considering all $N$ stages of the project decision and the process sliding window, there exists at least one path where:

$$R^*_{minacc}(N) = \sum_{i=N-W}^{N} r^*_{P_i}(d). \tag{5.1.8}$$

Among the set of cumulative sums $d^{(N-W)}$ at least one path of the cumulative sums exists that satisfies the equation 5.1.8.

Figure 5.2 shows the hypothetical value of the Bayes rule to calculate the metric of minimum risk management project for each scenario. In this case, we took into consideration the cumulative sums of $N$ stages, i.e. $W = 0$.

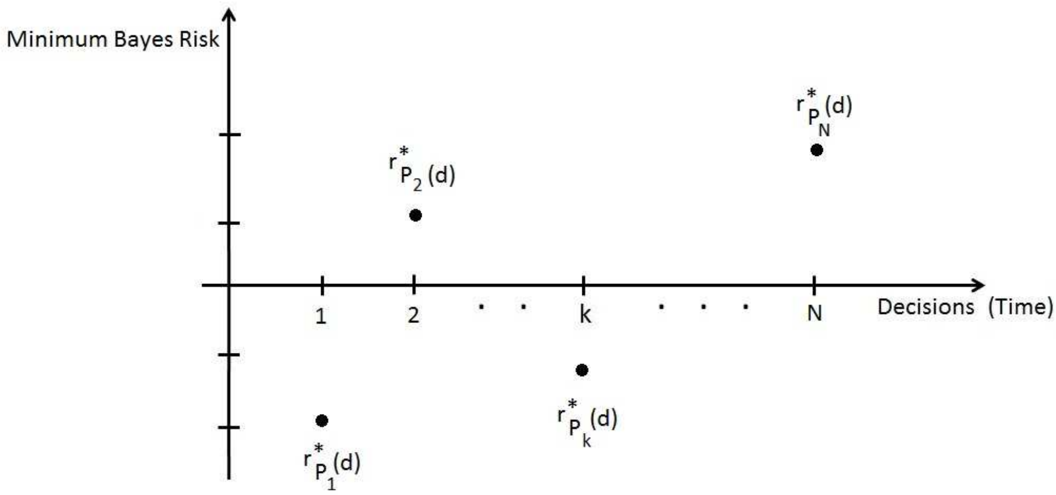


*Figure 5.2: Policy of Minimal Risks of the Project: Calculation of the Metrics.*

Therefore there is an association between the sequence of lower risk and the sequence of actions that minimize the risk and therefore the optimal strategy. As we have a guarantee for each parcel of the metrics proposed by equation 5.1.8, we are sure the sum of minimal risk will be the lowest of the possible sums. Thus, the ideal policy is defined for the project:

$$S_P^*(N,j) = \prod_{i=1}^{N} S_P^*(i,j) = S_P^*(1,j) \times S_P^*(2,j) \times \cdots \times S_P^*(N,j). \tag{5.1.9}$$

This proposed solution can be presented as a solution of a maximization problem. Whereas the states of nature may change over time, the decisions may, as well. The time interval $(t_0, t_1)$ represents the runtime of the project. Thus:

$$\max_{\{d,t\}} \sum_{t_0}^{t_1} \sum_{\Theta} \sum_{\mathcal{P}} u(p)\pi(\theta) f[p|\theta(t), d(t)] = \max_{\{d\}}[-r_d] \tag{5.1.10}$$

subject to

$$\sum_{\mathcal{P}} f[p|\theta(t), d(t)] = 1; \quad f[p|\theta(t), d(t)] \geq 0 \ \ \forall p, \theta, d, t.$$

$$f[p|\theta(t), d(t)] = \sum_{\mathcal{X}} f[x|\theta(t)] f[p|\theta(t), d(t)] \ \ \forall p, \theta, d, t$$

$$\sum_{\mathcal{X}} f[x|\theta(t)] = 1 \quad f[x|\theta(t)] \geq 0 \ \ \forall x, \theta.$$

$$\sum_{\Theta} \pi(\theta(t)) = 1; \quad \pi(\theta(t)) \geq 0 \ \ \forall x, \theta.$$

$$f[p|\theta(t+1), d(t+1)] = g[p, \theta(t), x(t)]; \quad \forall \theta, d, x.$$

The following will be seen in a model of a case study using the principles set out in the proposal for optimal strategy.

## 5.2 Risk Management

It is important to note that success in project management is not the same as the success of the project.

The importance of this work is the focus on the tools of project control with a proper management of risks for successful project implementation. Other factors are important, however, in the surrounding context of risk management (Davies, 2002):

1. Adequacy of company-wide education on the concepts of risk management.

2. Maturity of an organization's processes for assigning ownership of risks.

3. Adequacy with which a visible risk register is maintained.
4. Adequacy of an up-to-date risk management plan.
5. Adequacy of documentation of organizational responsibilities for the project.
6. Maintenance of project (or project stage duration) as far below 3 years as possible (1 year is better).
7. Allowance of changes in scope only through a mature scope change control process.
8. Maintainance of the integrity of the performance measurement baseline.
9. Existence of an effective benefits delivery and management process that involves the mutual co-operation of project management and line management functions.
10. Portfolio and programme management practices that allow the enterprise to resource a suite of projects that are thoughtfully and dynamically matched to the corporate strategy and business objectives.
11. A suite of project, programme and portfolio metrics that provides direct "line of sight" feedback on current project performance, and anticipated future success, so that project, portfolio and corporate decisions can be aligned.
12. An effective means of "learning from experience" on projects that combines explicit knowledge with tacit knowledge in a way that encourages people to learn and to embed that learning into continuous improvement of project management processes and practices.

For the project management community, it is also important to make the distinction between project success (which cannot be measured until after the project is completed) and project performance (which can be measured during the life of the project). No system of project metrics is complete without both sets of measures (performance and success) and a means of linking them so as to assess the accuracy with which performance predicts success.

The Project Life Cycle is often described in terms of four phases, with terms such as conceptualization, planning, execution and termination according to Ward (1995).

The model of project execution strategy that includes execution is related critical success factors that are made in the project planning phase. Even though the project execution strategy does not help much to achieve the higher levels of project success, it is essential especially for project suppliers whose businesses are focused on project deliveries (Pulkkinen, 2005). Risk management should help to ensure compliance at all stages of the Project Life Cycle.

Success refers to how well the project is able to accomplish its goals. Each project stakeholder may have different and conflicting criteria for evaluating project's degree of success (Artto, 2008).

Based on Artto (2008), "Project strategy is a direction in a project that contributes to success of the project in its environment". We can conclude that the degree of independence of the project and the number of strong project stakeholders are important parameters in the project environment that can be used to explain different strategies in projects. Four different types of design strategies are related to the profiles of decision makers seen in the section 2.2.6: the obedient servant, the independent innovator, the flexible moderator, and the strong leader.

The four types of design strategy are explained in figure 5.3, in terms questions of direction and success of our strategy for defining a project.

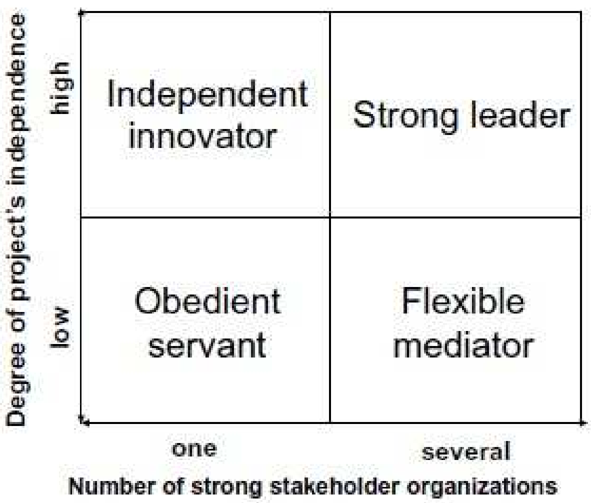


*Figure 5.3: Four Obvious Project Strategies Depending on Project's Independence and Number of Strong Project Stakeholder Organizations (Artto, 2008).*

### 5.2.1 Operational Systematics of Risk Management

This section will focus on scientific efforts to systematize the identification, quantification, qualification and analysis of risks, in the context of monitoring and control of risks in complex projects, where a manager is directly involved, culminating in a proposal for the completion of a Risk Action Plan.

The selection process and development in the treatment of risk, according to Cooper (2005) involves the following steps:

1. Risk Identification.

   Risk Identification determines what can happen that could affect the objectives of the project. The process of risk identification must be comprehensive.

   The risks that are not identified represent a threat to project success. Information used in the risk identification process may include historical data, theoretical analysis, empirical data and analysis, informed opinions of the project team and other experts, and the concerns of stakeholders.

2. Estimate Risk.

   The purpose of risk estimate is to develop priorities for further treatment.

   - The systematic use of available information permits risk estimate by identifying the frequency and intensity of events.
   - Risk assessment is the process of identifying the significance of risk.

3. Development of Risk.

   The processes described here are to identify risks in the systems, subsystems and elements of the project:

   - Use a semi-quantitative approach to assess the likelihood of each risk element, and its consequences.
   - Associate the probability of occurrence of the event with its consequences.

4. Risk Treatment.

   Risk treatment determines actions necessary to respond to identified risks so as to reduce their consequences. Treatment involves:

   - risk prevention (including risk avoidance)
   - impact mitigation
   - risk mitigation
   - insurance, and
   - risk retention.
   - Identification of the options for reducing the likelihood or consequences of each extreme, high or medium risk.
   - Determine of the potential benefits and costs of the options.
   - Select of the best options for the project.
   - Development and implementation of detailed Risk Action Plans.
   - Inclusion of appropriate provisions in project budgets.

5. Monitoring and Control Risk.

   The monitoring and control of risk is to facilitate better risk management and promote continuous improvement.

   - Implement a review process and tracking of risks should be part of regular cycle management.
   - Major reviews should be taken at significant phases and milestones of the project.

## 5.3 Decision Theory: Process Modeling

This case study will evaluate the risk management in the operation of a nuclear power station using the constructs of Decision Theory, where situations are simulated by complex decisions that can be chosen using the metric of lower risk. It should be remembered that some of the biggest accidents worldwide have happened in the context of human decisions-making.

### 5.3.1 Main Accidents in Nuclear Power Plants around the World

In the world, Europe is the region that most uses nuclear energy as a source of electric power generation, about 30%. North America uses 17%. The three countries that respond for 60% of installed capacity in nuclear power plants are Japan, France and the United States. In France about 80% of the electricity is generated from nuclear sources, and in Japan about 30%. However, despite levels of security applied to the operations of nuclear plants and the benefits they provide, there are always risks of accidents caused by natural hazards such as earthquakes, tidal waves, tsunamis, and by human error and equipment failure. Major accidents involving nuclear power plants in the world were:

A) Chernobyl in Ukraine in 1986

In 1986, operators of the Chernobyl nuclear plant in the Ukraine, conducted an experiment with reactor 4. The original intention was to observe the behavior of the reactor when used with low energy levels. But for the test to be possible, those responsible for the unit would have to break a number of essential safety rules. It was then that a huge nuclear disaster in Eastern Europe was destined. Among other errors, the employees involved in the episode interrupted the flow of the hydraulic system that controlled the temperature of the reactor. Thus, even when operating at lower capacity, the reactor began a process of overheating which was unable to be reversed. In just few moments an explosion was caused in the reactor which was rich in Cesium-137, a chemical element of potent radioactive power. In this event, the Chernobyl plant released a lethal amount of radioactive material that contaminated a kilometer of the atmosphere in the region. In comparison, the radioactive material disseminated at that time was frighteningly four hundred times greater than the bombs used in the bombing of Hiroshima and Nagasaki at the end of World War II. Finally, a cloud of radioactive material contaminated the Ukrainian city of Pripyat. So there are two official reasons that caused the accident: human error (1986) and flaw in the design of the reactor (1991). The accident at Chernobyl was grade 7 on the INES scale (International Nuclear Event Scale).

The Kyshtym disaster was a radiation contamination incident that occurred on $29^{th}$ of September 1957 at Mayak, a nuclear fuel reprocessing plant in Russia (then a part of the Soviet Union). It measured as a Level 6 disaster on the International Nuclear Event Scale, making it the third most serious nuclear accident ever recorded (after the Chernobyl disaster, and Fukushima Daiichi nuclear disaster, both Level 7 on the INES scale). The event occurred in the town of Ozyorsk, a closed city built around the Mayak plant. Since Ozyorsk/Mayak (also known as Chelyabinsk-40 and Chelyabinsk-65) was not marked on maps, the disaster was named after Kyshtym, the nearest known town.

B) Mayak in Russia in 1957

Mayak was a factory processing nuclear material whose explosion occurred due to a failure in the cooling system of a tank that stores thousands of tons of nuclear waste, which caused an explosion of force corresponding to 75 tons of TNT. In addition to the explosion, there were other disasters at Mayak, such as radioactive waste being poured directly into the Techa River, used as a water source for thousands of people. It was the first major nuclear accident and one of the largest, along with that of Chernobyl, which occurred in the Soviet Union. It measured as a level 6 disaster on the International Nuclear Event Scale, making it the third most serious nuclear accident ever recorded (after the Chernobyl disaster, and Fukushima Daiichi nuclear disaster.

The cause of the accident was attributed to equipment failure.

C) Three Mile Island in the U.S. in 1979

The accident at Three Mile Island was a partial nuclear meltdown in Unit 2 nuclear power plant at Three Mile Island in Dauphin County near Harrisburg. It was the most significant accident in the history of the industry's commercial nuclear power generation in America, and resulted in the release of up to 481 PBq of radioactive gases and less than 740 GBq of iodine-131 which is particularly dangerous. The accident began at 4 am on Wednesday, March 28, 1979, with gaps in the non-nuclear secondary system, followed by a relief valve pilot operated primary system that had been left open, allowing large amounts of coolant to escape. The mechanical failures were created by the initial failure of the reactor operators to recognize the situation as an accident of coolant loss. This was due to improper training and human factors and industrial design errors related to the presence of ambiguous indicators in the control room interface user of the plant.

The cause of the accident was attributed to equipment failure, followed by human error. The accident at Three Mile Island, was at level 5 of the INES (International Nuclear Event Scale).

D) Fukushima in Japan in 2011

One day after the great earthquake and tsunami that struck Japan, the Fukushima nuclear power plant suffered an explosion due to the earthquake of 8.9 on the Richter scale that shook the Tohoku region. Then there was a tsunami that struck the province causing substantial damage. The earthquake shook the structure of the Fukushima nuclear power plant, causing a serious nuclear accident.

The power plant consists of six boiling water reactors separately maintained by the Tokyo Electric Power Company (TEPCO). The reactors 4, 5 and 6 had been closed for maintenance before the earthquake. The remaining reactors were shut down automatically after the earthquake and emergency generators were started to keep the water pumps needed to cool them. The plant was protected by a levee designed to withstand an earthquake of 5.7 meters in height, but about 15 minutes after the quake was hit by a wave of 14 meters, which easily topped the seawall. The entire plant, including the generator of low altitude, was flooded. As a result, the emergency generators were turned off and the reactor began to overheat due to natural decay of the nuclear fuel contained in them. The damage caused by flood and earthquake prevented the arrival of assistance that needed to be brought in from elsewhere.

Evidence pointed to a partial melting of the reactor core in 1, 2 and 3; explosions destroyed the top coat hydrogen of buildings housing the reactors 1, 3 and 4, an explosion damaged the containment into the reactor 2, and multiple fires broke out in the reactor 4. In addition, the fuel rods stored in spent fuel pools of 1-4 units began to overheat the water levels in abandoned swimming pools. Fears of radiation leaks prompted the evacuation of a 20 km radius around the plant. Factory workers were exposed to radiation and were temporarily evacuated at various times. The Japanese authorities have designated the magnitude of the danger in reactors 1, 2 and 3 at level 7 in Section 7 of the International Scale of Nuclear Accidents (INES).

### 5.3.2 Case Study: Risk Management in the Operation of a Nuclear Power Station

The modeling process is important to simulate real-life situations can occur and prepare project managers for a real situation.

A case study of management and operation of a nuclear power plant, with the modeling done using Decision Theory to a single-stage decision ($k = 1$), is shown below.

i) *Payoffs*

The payoff vector will be considered composed of four attributes, in this case, as placed in order of importance by the EAM (initials of the name) decision-maker, whose profile is the executive manager of a private company:

$$\vec{p} = \{p_i\} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \tag{5.3.1}$$

where $p_i \in \{0, 1\}, i = 1, \ldots, 4$, with

$p_1$ — represents the human aspects, indicating the morbi-mortality of the disaster, with the number of deaths. Two values can be assumed for $p_1 = 0$, represents a unacceptable situation, which indicates a high number of deaths and injuries, in contrast, $p_1 = 1$, indicates a significantly better situation, with a low number of deaths and injuries.

$p_2$ — represents the environment, referring to the environmental impact, along with the changes done in the bio-diversity of the environment, measured in the affected area in $km^2$. Two values can be taken for $p_2 = 0$, represents a bad situation, for which reason a large area can be affected, unlike $p_2 = 1$, indicates a significantly better situation, with a small affected area.

$p_3$ — represents the physical structure of the region, indicating the cost of property damaged or destroyed materials, measured in dollars. Two values can be taken for $p_3 = 0$, represents a major financial loss, by contrast $p_3 = 1$, indicates a significantly better situation, with a small financial loss.

$p_4$ — represents investment in solutions to minimize the impacts of destruction, indicating the cost to implement an alert system for people with training, measured in dollars. Two values can be assumed for $p_4 = 0$, represents a small financial investment, in contrast $p_4 = 1$, represents a considerable financial investment.

Performing the ordering of the payoffs, we can see in Table 5.1:

*Table 5.1: Possible Deterministic Consequences.*

| | $\vec{p}^{\,1}$ | $\vec{p}^{\,2}$ | $\vec{p}^{\,3}$ | $\vec{p}^{\,4}$ | $\vec{p}^{\,5}$ | $\vec{p}^{\,6}$ | $\vec{p}^{\,7}$ | $\vec{p}^{\,8}$ | $\vec{p}^{\,9}$ | $\vec{p}^{\,10}$ | $\vec{p}^{\,11}$ | $\vec{p}^{\,12}$ | $\vec{p}^{\,13}$ | $\vec{p}^{\,14}$ | $\vec{p}^{\,15}$ | $\vec{p}^{\,16}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $p_2$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| $p_3$ | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| $p_4$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |

ii) *States of Nature*

The representative set of states of nature reflect three levels of scenarios:

$\Theta = \{\theta_1, \theta_2, \theta_3\}$, where:

$\theta_1$ — represents a set of attributes related to an excellent state of nature: favorable climatic conditions, without predicted changes in the weather forecast, a growing country with strong economic power in public investment, while fully meeting the demand of electricity. Human resources working in the plant as well trained and qualified. Measuring equipment calibrated and in good condition.

$\theta_2$ — represents a set of attributes related to a stable state of nature: regular weather conditions, economic situation of the country stable and proper operation of the energy matrix.

$\theta_3$ — represents a set of unfavorable situations, with unfavorable climatic conditions, predictions of earthquakes and tsunamis, the economic situation of the country in crisis, with an increase in energy deficit Human resources working in the plant without adequate training. Measuring equipment without calibration subject to measurement errors.

iii) *Action Set*

$\mathcal{A} = \{a_1, a_2, a_3\}$

- $a_1$ — Execute investments to expand the power plant, best qualified employees with increased training, planning to expand the medium and long term, improve customer relationships.
- $a_2$ — Control the supply of electricity and maintain the power plant without major investments.
- $a_3$ — Interrupt the supply of electricity, the contingency and accident management plans put in place, evacuate the surrounding population.

iv) *The Decision Maker Preferences - The Utility Function*

System for Educing Preferences - SEP as developed by Albuquerque (2011), was used the utility function of the manager for the problem in question. The points obtained are shown in table 5.2:

The vectors are placed in lexicographic order, since the order of priorities have been established by the decision maker, therefore:

$$\vec{p}^{\,1} \prec \vec{p}^{\,2} \prec \vec{p}^{\,3} \prec \cdots \prec \vec{p}^{\,16}$$

So the worst consequence is $u(\vec{p}^{\,1}) = 0$ and the best, $u(\vec{p}^{\,16}) = 1$.

Intermediate values are determined by talking the decision maker through a lottery with the objective of determining $\lambda_j$, ie:

$$\vec{p}^{\,j}\text{with probability } 1 \sim \begin{cases} \vec{p}^{\,16} & \text{with probability} \quad \lambda_j \\ \vec{p}^{\,1} & \text{with probability} \quad 1-\lambda_j \end{cases}$$

Through the answer given by the decision maker to the SEP system, which draws up scenarios like the lottery as above, where the range of the decision maker's utility function is in the interval $[0,1]$, so for each scenario it is given the value of $\lambda_j$ is given for each proposed situation on a scale of $\vec{p}^{\,1}$ to $\vec{p}^{\,16}$, which thereby determined how to proceed according to the preferences of the decision-maker.

The points of the curve of the EAM decision-maker's utility function is shown in the table 5.2:

*Table 5.2: Decision Maker's Utility Function.*

| Utility | $\vec{p}^{\,1}$ | $\vec{p}^{\,2}$ | $\vec{p}^{\,3}$ | $\vec{p}^{\,4}$ | $\vec{p}^{\,5}$ | $\vec{p}^{\,6}$ | $\vec{p}^{\,7}$ | $\vec{p}^{\,8}$ |
|---|---|---|---|---|---|---|---|---|
| $u(p)$ | 0,0000 | 0,0235 | 0,0269 | 0,0235 | 0,0336 | 0,0303 | 0,0303 | 0,0336 |

| Utility | $\vec{p}^{\,9}$ | $\vec{p}^{\,10}$ | $\vec{p}^{\,11}$ | $\vec{p}^{\,12}$ | $\vec{p}^{\,13}$ | $\vec{p}^{\,14}$ | $\vec{p}^{\,15}$ | $\vec{p}^{\,16}$ |
|---|---|---|---|---|---|---|---|---|
| $u(p)$ | 0,0572 | 0,0572 | 0,0572 | 0,0572 | 0,1515 | 0,1515 | 0,1515 | 1,00 |

The curve of figure 5.4 shows the points of the table 5.2, which represents the curve of the utility function of the decision-maker.

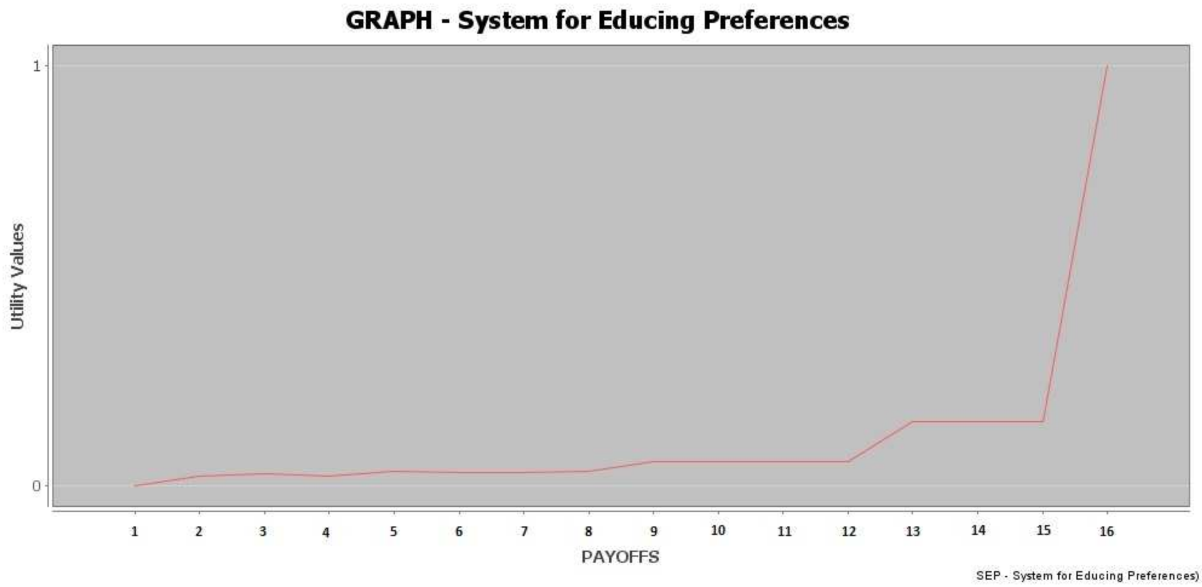


*Figure 5.4: Dispersion of the Manager's Utility Function EAM.*

v) *Consequence Functions*

The consequence function result is the probability of getting a payoff $p$, since the nature decides a state $\theta$ and the decision maker chooses action $a$. The function can therefore be estimated from the experience of specialists in the process of eduction or by means of historical data. Binomial distribution was used to simulate the data for purposes of calculation of the probabilities of the consequence function, according to Table 5.3:

*Table 5.3: Consequence Function $P(p|\theta, a)$.*

| $(\theta, a)$ | $\vec{p}^{\,1}$ | $\vec{p}^{\,2}$ | $\vec{p}^{\,3}$ | $\vec{p}^{\,4}$ | $\vec{p}^{\,5}$ | $\vec{p}^{\,6}$ | $\vec{p}^{\,7}$ | $\vec{p}^{\,8}$ | $\vec{p}^{\,9}$ | $\vec{p}^{\,10}$ | $\vec{p}^{\,11}$ | $\vec{p}^{\,12}$ | $\vec{p}^{\,13}$ | $\vec{p}^{\,14}$ | $\vec{p}^{\,15}$ | $\vec{p}^{\,16}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(\theta_1, a_1)$ | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,02 | 0,01 | 0,03 | 0,09 | 0,19 | 0,29 | 0,27 | 0,12 |
| $(\theta_1, a_2)$ | 0,00 | 0,00 | 0,00 | 0,00 | 0,01 | 0,02 | 0,06 | 0,12 | 0,18 | 0,21 | 0,19 | 0,13 | 0,06 | 0,02 | 0,00 | 0,00 |
| $(\theta_1, a_3)$ | 0,00 | 0,00 | 0,02 | 0,06 | 0,13 | 0,19 | 0,21 | 0,18 | 0,12 | 0,06 | 0,02 | 0,01 | 0,00 | 0,00 | 0,00 | 0,00 |
| $(\theta_2, a_1)$ | 0,00 | 0,00 | 0,00 | 0,01 | 0,03 | 0,06 | 0,12 | 0,18 | 0,20 | 0,18 | 0,12 | 0,06 | 0,02 | 0,01 | 0,00 | 0,00 |
| $(\theta_2, a_2)$ | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,01 | 0,02 | 0,05 | 0,11 | 0,18 | 0,23 | 0,21 | 0,13 | 0,05 | 0,01 |
| $(\theta_2, a_3)$ | 0,00 | 0,02 | 0,06 | 0,13 | 0,19 | 0,21 | 0,18 | 0,11 | 0,06 | 0,02 | 0,01 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| $(\theta_3, a_1)$ | 0,12 | 0,27 | 0,29 | 0,19 | 0,09 | 0,03 | 0,01 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| $(\theta_3, a_2)$ | 0,04 | 0,13 | 0,23 | 0,25 | 0,19 | 0,10 | 0,04 | 0,01 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| $(\theta_3, a_3)$ | 0,01 | 0,05 | 0,13 | 0,21 | 0,23 | 0,18 | 0,11 | 0,05 | 0,02 | 0,01 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |

Utility of Consequence Function:

$$u(P(p|\theta, a)) = \sum_{p} v(p)P(p|\theta, a) \tag{5.3.2}$$

The values $v(p)$ are obtained from the utility function of payoffs in table 5.2, while the values of $P(p|\theta, a)$ are taken from table 5.3, resulting in table 5.4.

*Table 5.4: Utility of Consequence Function $u(P(p|\theta, a))$.*

| State of Nature, Space Actions | $u(P(p\|\theta, a))$ |
|---|---|
| $(\theta_1, a_1)$ | 0,303 |
| $(\theta_1, a_2)$ | 0,298 |
| $(\theta_1, a_3)$ | 0,472 |
| $(\theta_2, a_1)$ | 0,403 |
| $(\theta_2, a_2)$ | 0,190 |
| $(\theta_2, a_3)$ | 0,380 |
| $(\theta_3, a_1)$ | 0,096 |
| $(\theta_3, a_2)$ | 0,146 |
| $(\theta_3, a_3)$ | 0,250 |

Calculating the Loss Function $L(\theta, a)$, shown in table 5.5, which is the negative consequence of the utility function in table 5.4:

*Table 5.5: Loss Function $L(\theta, a)$.*

| State of Nature, Space Actions | $L(\theta, a)$ |
|---|---|
| $(\theta_1, a_1)$ | $-0,303$ |
| $(\theta_1, a_2)$ | $-0,298$ |
| $(\theta_1, a_3)$ | $-0,472$ |
| $(\theta_2, a_1)$ | $-0,403$ |
| $(\theta_2, a_2)$ | $-0,190$ |
| $(\theta_2, a_3)$ | $-0,380$ |
| $(\theta_3, a_1)$ | $-0,096$ |
| $(\theta_3, a_2)$ | $-0,146$ |
| $(\theta_3, a_3)$ | $-0,250$ |

vi) *Set of Observations*

$\mathcal{X} = \{x_1, x_2, x_3\}$

Decision Maker observes the set of $\mathcal{X}$ to choose the best action, according to the following observations:

- $x_1$ — periodic measurements indicate that the meteorological data are normal, in line with the optimistic scenario $\theta_1$.
- $x_2$ — the measurements indicate a moderate weather scenario. Is an indication that the state of nature will have a value $\theta_2$.

- $x_3$ — the measurements performed show indications of abnormal weather conditions, showing a warning scenario, pessimistic or negative similar to $\theta_3$.

vii) *Likelihood Function*

$P(x|\theta)$ is the function that maps the set of observations seen by the decision maker with all the states of nature, conditionally. The likelihood function is shown in table 5.6, hence:

*Table 5.6: Likelihood Function $P(x|\theta)$.*

| observations | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|---|---|---|---|
| $x_1$ | 0,70 | 0,10 | 0,05 |
| $x_2$ | 0,20 | 0,85 | 0,20 |
| $x_3$ | 0,10 | 0,05 | 0,75 |

viii) *Function Risk*

From the calculation of the loss function and values of the likelihood function, one can calculate the risk:

$$R_d = \sum_x L(\theta, d(x))P(x|\theta) \tag{5.3.3}$$

Considering the possible combination of decision rules, we obtain the rule in Table 5.7:

*Table 5.7: Decision Making Rules.*

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_1$ | $a_1$ | $a_2$ | $a_3$ |
| $d_2$ | $a_1$ | $a_3$ | $a_2$ |
| $d_3$ | $a_2$ | $a_1$ | $a_3$ |
| $d_4$ | $a_2$ | $a_3$ | $a_1$ |
| $d_5$ | $a_3$ | $a_1$ | $a_2$ |
| $d_6$ | $a_3$ | $a_2$ | $a_1$ |
| $d_7$ | $a_1$ | $a_1$ | $a_2$ |
| $d_8$ | $a_1$ | $a_1$ | $a_3$ |
| $d_9$ | $a_2$ | $a_2$ | $a_1$ |

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_{10}$ | $a_2$ | $a_2$ | $a_3$ |
| $d_{11}$ | $a_3$ | $a_3$ | $a_1$ |
| $d_{12}$ | $a_3$ | $a_3$ | $a_2$ |
| $d_{13}$ | $a_2$ | $a_1$ | $a_1$ |
| $d_{14}$ | $a_3$ | $a_1$ | $a_1$ |
| $d_{15}$ | $a_1$ | $a_2$ | $a_2$ |
| $d_{16}$ | $a_3$ | $a_2$ | $a_2$ |
| $d_{17}$ | $a_1$ | $a_3$ | $a_3$ |
| $d_{18}$ | $a_2$ | $a_3$ | $a_3$ |

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_{19}$ | $a_1$ | $a_2$ | $a_1$ |
| $d_{20}$ | $a_1$ | $a_3$ | $a_2$ |
| $d_{21}$ | $a_2$ | $a_3$ | $a_2$ |
| $d_{22}$ | $a_2$ | $a_1$ | $a_2$ |
| $d_{23}$ | $a_3$ | $a_1$ | $a_3$ |
| $d_{24}$ | $a_3$ | $a_2$ | $a_3$ |
| $d_{25}$ | $a_1$ | $a_1$ | $a_1$ |
| $d_{26}$ | $a_2$ | $a_2$ | $a_2$ |
| $d_{27}$ | $a_3$ | $a_3$ | $a_3$ |

Thus, considering the set of decision making rules one can calculate the risk function in table 5.8, for each value of $\theta$, considering the loss function $L(\theta)$ and the likelihood function, tables 5.5 and 5.6.

*Table 5.8: Risk Function $R_d$.*

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_1$ | $-0,183$ | $-0,094$ | $-0,029$ |
| $d_2$ | $-0,180$ | $-0,038$ | $-0,027$ |
| $d_3$ | $-0,094$ | $-0,055$ | $-0,028$ |
| $d_4$ | $-0,074$ | $-0,040$ | $-0,024$ |
| $d_5$ | $-0,079$ | $-0,051$ | $-0,026$ |
| $d_6$ | $-0,062$ | $-0,093$ | $-0,024$ |
| $d_7$ | $-0,221$ | $-0,053$ | $-0,026$ |
| $d_8$ | $-0,218$ | $-0,049$ | $-0,028$ |
| $d_9$ | $-0,079$ | $-0,100$ | $-0,024$ |
| $d_{10}$ | $-0,058$ | $-0,099$ | $-0,029$ |
| $d_{11}$ | $-0,057$ | $-0,033$ | $-0,025$ |
| $d_{12}$ | $-0,039$ | $-0,036$ | $-0,027$ |
| $d_{13}$ | $-0,114$ | $-0,055$ | $-0,023$ |
| $d_{14}$ | $-0,097$ | $-0,048$ | $-0,023$ |

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_{15}$ | $-0,185$ | $-0,097$ | $-0,026$ |
| $d_{16}$ | $-0,044$ | $-0,096$ | $-0,027$ |
| $d_{17}$ | $-0,178$ | $-0,034$ | $-0,029$ |
| $d_{18}$ | $-0,135$ | $-0,039$ | $-0,029$ |
| $d_{19}$ | $-0,203$ | $-0,095$ | $-0,024$ |
| $d_{20}$ | $-0,180$ | $-0,038$ | $-0,027$ |
| $d_{21}$ | $-0,056$ | $-0,042$ | $-0,027$ |
| $d_{22}$ | $-0,096$ | $-0,058$ | $-0,026$ |
| $d_{23}$ | $-0,077$ | $-0,048$ | $-0,028$ |
| $d_{24}$ | $-0,041$ | $-0,092$ | $-0,029$ |
| $d_{25}$ | $-0,238$ | $-0,050$ | $-0,023$ |
| $d_{26}$ | $-0,061$ | $-0,120$ | $-0,027$ |
| $d_{27}$ | $-0,037$ | $-0,032$ | $-0,029$ |

ix) *Prior Knowledge* $\pi(\theta)$

Because the experience of the EAM manager was based on weather conditions around the plant, ie, 85% of the time the weather conditions are good or reasonable. The prior probabilities $\pi(\theta)$ were educed according to table 5.9:

*Table 5.9: Prior Probabilities.*

| $\theta$ | $\pi(\theta)$ |
|---|---|
| $\theta_1$ | 0,55 |
| $\theta_2$ | 0,30 |
| $\theta_3$ | 0,15 |

x) *Bayes Decision Rule*

With the information of the prior probability distribution, we calculate the risk of Bayes by:

$$r_d = \sum_{\theta} \pi(\theta) R_d(\theta) \tag{5.3.4}$$

The Risk of Bayes is shown in table 5.10.

*Table 5.10: Bayes Risk $r_d$.*

| decisions making rules | $x_1$ | $x_2$ | $x_3$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $r_d$ |
|---|---|---|---|---|---|---|---|
| $d_1$ | $a_1$ | $a_2$ | $a_3$ | $-0,183$ | $-0,094$ | $-0,029$ | $-0,133$ |
| $d_2$ | $a_1$ | $a_3$ | $a_2$ | $-0,180$ | $-0,038$ | $-0,027$ | $-0,115$ |
| $d_3$ | $a_2$ | $a_1$ | $a_3$ | $-0,094$ | $-0,055$ | $-0,028$ | $-0,072$ |
| $d_4$ | $a_2$ | $a_3$ | $a_1$ | $-0,074$ | $-0,040$ | $-0,024$ | $-0,056$ |
| $d_5$ | $a_3$ | $a_1$ | $a_2$ | $-0,079$ | $-0,051$ | $-0,026$ | $-0,063$ |
| $d_6$ | $a_3$ | $a_2$ | $a_1$ | $-0,062$ | $-0,093$ | $-0,024$ | $-0,065$ |
| $d_7$ | $a_1$ | $a_1$ | $a_2$ | $-0,221$ | $-0,053$ | $-0,026$ | $-0,141$ |
| $d_8$ | $a_1$ | $a_1$ | $a_3$ | $-0,218$ | $-0,049$ | $-0,028$ | $-0,139$ |
| $d_9$ | $a_2$ | $a_2$ | $a_1$ | $-0,079$ | $-0,100$ | $-0,024$ | $-0,077$ |
| $d_{10}$ | $a_2$ | $a_2$ | $a_3$ | $-0,058$ | $-0,099$ | $-0,029$ | $-0,066$ |
| $d_{11}$ | $a_3$ | $a_3$ | $a_1$ | $-0,057$ | $-0,033$ | $-0,025$ | $-0,045$ |
| $d_{12}$ | $a_3$ | $a_3$ | $a_2$ | $-0,039$ | $-0,036$ | $-0,027$ | $-0,036$ |
| $d_{13}$ | $a_2$ | $a_1$ | $a_1$ | $-0,114$ | $-0,055$ | $-0,023$ | $-0,083$ |
| $d_{14}$ | $a_3$ | $a_1$ | $a_1$ | $-0,097$ | $-0,048$ | $-0,023$ | $-0,071$ |
| $d_{15}$ | $a_1$ | $a_2$ | $a_2$ | $-0,185$ | $-0,097$ | $-0,026$ | $-0,135$ |
| $d_{16}$ | $a_3$ | $a_2$ | $a_2$ | $-0,044$ | $-0,096$ | $-0,027$ | $-0,057$ |
| $d_{17}$ | $a_1$ | $a_3$ | $a_3$ | $-0,178$ | $-0,034$ | $-0,029$ | $-0,112$ |
| $d_{18}$ | $a_2$ | $a_3$ | $a_3$ | $-0,135$ | $-0,039$ | $-0,029$ | $-0,091$ |
| $d_{19}$ | $a_1$ | $a_2$ | $a_1$ | $-0,203$ | $-0,095$ | $-0,024$ | $-0,144$ |
| $d_{20}$ | $a_1$ | $a_3$ | $a_2$ | $-0,180$ | $-0,038$ | $-0,027$ | $-0,115$ |
| $d_{21}$ | $a_2$ | $a_3$ | $a_2$ | $-0,056$ | $-0,042$ | $-0,027$ | $-0,048$ |
| $d_{22}$ | $a_2$ | $a_1$ | $a_2$ | $-0,096$ | $-0,058$ | $-0,026$ | $-0,074$ |
| $d_{23}$ | $a_3$ | $a_1$ | $a_3$ | $-0,077$ | $-0,048$ | $-0,028$ | $-0,061$ |
| $d_{24}$ | $a_3$ | $a_2$ | $a_3$ | $-0,041$ | $-0,092$ | $-0,029$ | $-0,055$ |
| $d^*_{25}$ | $a_1$ | $a_1$ | $a_1$ | $-0,238$ | $-0,050$ | $-0,023$ | $-0,150$ |
| $d_{26}$ | $a_2$ | $a_2$ | $a_2$ | $-0,061$ | $-0,120$ | $-0,027$ | $-0,068$ |
| $d_{27}$ | $a_3$ | $a_3$ | $a_3$ | $-0,037$ | $-0,032$ | $-0,029$ | $-0,034$ |

Thus, the decision rule that minimizes the Bayes risk is the one that has the maximum utility. That rule is $d^*_{25} = -0,150$. It may be noted that this table 5.10 represents the lowest risk among all calculated, corresponding to the following situation shown in table 5.11:

*Table 5.11: Choose the Best Decision Making Rule.*

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d^*_{25}$ | $a_1$ | $a_1$ | $a_1$ |

Thus, the best action corresponding to "Execute investments to expand the power plant, best qualified employees with increased training, planning to expand the medium and long term, improve customer relationships". The action $a_1$ should be chosen independent of observation $\mathcal{X}$ made by the decision-maker. However, this choice is characterized by the profile by which the project manager was chosen. This case, the decision-maker has a profile of risk propensity, so that decisions are more daring. From the data educed the decision-maker through the SEP system is plotted in figure 5.4 the corresponding curve. It is observed by the curvature of the graph that the decision-maker has the characteristics of risk-prone. The layout of the curve is roughly like the one shown in figure 2.5 in subsection 2.2.6.

One clear fact is that despite an observation $x_3$ where the scenario consists of a set of unfavorable situations, with unfavorable climatic conditions, predictions of earthquakes and tsunamis, the economic situation of the country in crisis, with an increase in energy deficit. The decision-maker would make the decision to maintain investment in the plant. An alternative would be the project manager to contract an insurer covering any damage and to reduce the financial impacts of the project, according to subsection 4.1.2.

A suggestion to improve the overall project would make use of incentive clauses of the Principal-Agent theory. For this, the payoffs would be set by creating favorable conditions for the project.

In sequential decision-making the next decision will take into account the new scenario $(k \geq 2)$ that can be recalculated for all risks associated with the new situation.

# 6 A SACRED ART MUSEUM PROJECT

By the fact that Pernambuco has its coastline open to the Atlantic Ocean, it was the destination of many cultural and artistic missions such as those of Maurício de Nassau and the Conde da Boa Vista. The former brought to America the evolution of human thought and utility no longer coated with an economy of conquest, but backed by a revolutionary design with local and immediate interests. The latter is a living reflection of the industrial revolution. The early region of Pernambuco is of an intellectual subsistence which has been the hallmark of his civic and cultural development. It was home to serious and intelligent discussions, as well as being the birthplace of grandiose dreams of libertarian progressive movements in the nineteenth century.

Pernambuco had in Tobias Barreto and Silvio Romero, the so-called School of Recife, a true foundational of philosophical thought of a high category, as well as Gilberto Freyre, who crystallized sociological thought in Brazil. The state welcomed the masterpieces of Portuguese Baroque art as found in the Franciscan convents, as well as Vautier for French neoclassical architecture and Luiz Nunes Silveira for modern Brazilian architecture.

Pernambuco, the land of poets and revolutionary thinkers, always in the forefront of political thought, has a historical, cultural and sacred art sufficient for creating a museum of sacred art in the State of Pernambuco.

Sacred art is defined as an authentic form of work of art and completely directed the sacredness of the rite that is destined.

## 6.1 Objectives for the Project of Sacred Art in the State of Pernambuco

### 6.1.1 General Objectives

Although there is Maspe - Sacred Art Museum in the State of Pernambuco, the premise was to provide the State of Pernambuco through the execution of a project, a different cultural space for sacred art, as well as encourage capabilities already available in the state, both because of Pernambuco lives with a growing cultural potential in the arts, literature, in music, and cuisine and also has a large collection of religious art that could constitute a cultural, artistic and historical center in the State of Pernambuco through the establishment of a Museum of Sacred Art.

### 6.1.2 Specific Objectives

The project itself includes the following specific objectives:

1. To develop and implement the project and the installation of a Museum of Sacred Art in Pernambuco State, including the development of a space for the preservation of the local culture.

2. Establish a permanent agenda of cultural events linked to religious art and culture in general. Can be said that religious art is one that reflects the religious life of the artist.

3. Create an environment conducive to the study of arts, including art history.

4. Contribute to the improvement of research and knowledge of the history of Pernambuco.

## 6.2 Case Study: Risk Management to Implement a Project for a Museum of Sacred Art in the State of Pernambuco

i) *Payoffs*

A vector composed of four variables will be considered the payoff, in this case, ranked by degree of importance by a specialist:

$$\vec{p} = \{p_i\} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix} \tag{6.2.1}$$

where $p_i \in \{0, 1\}, i = 1, \ldots, 4$, with

$p_1$ — represents the scope of the deployment of a project for a museum of sacred art. Using a dichotomous scale for the payoffs, two values are obtained: for $p_1 = 0$, represents a bad situation, indicating that the project scope has not been reached, in contrast, $p_1 = 1$, indicate a significantly better situation, indicating that the scope of the project was fully reached.

$p_2$ — represents the total project cost estimated at 10 million dollars. Two values can be taken for $p_2 = 0$, represents a bad situation, therefore the projected costs were extrapolated, unlike $p_2 = 1$, which indicates a significantly better situation, with the implementation of the project within the budgeted costs.

$p_3$ — represents the final quality of the project, assuming two possible options: for $p_4 = 0$, indicating that the project was completed outside the established quality standards, in contrast to $p_4 = 1$, which indicates that the project was completed within the established quality standards.

$p_4$ — represents the time of execution of the project. Two values can be taken for $p_3 = 0$, indicating that the implementation of the project went beyond the deadline, by in contrast to $p_3 = 1$, which indicates that the project was completed on time.

To define the order of importance of the attributes performed by ABM (initials of the name) decision-maker, whose profile is close to a project manager, Hasse's algorithm was observed, along with procedures established in Wanderley (2008). The diagram is shown in Figure 6.1 for the case of four attributes.

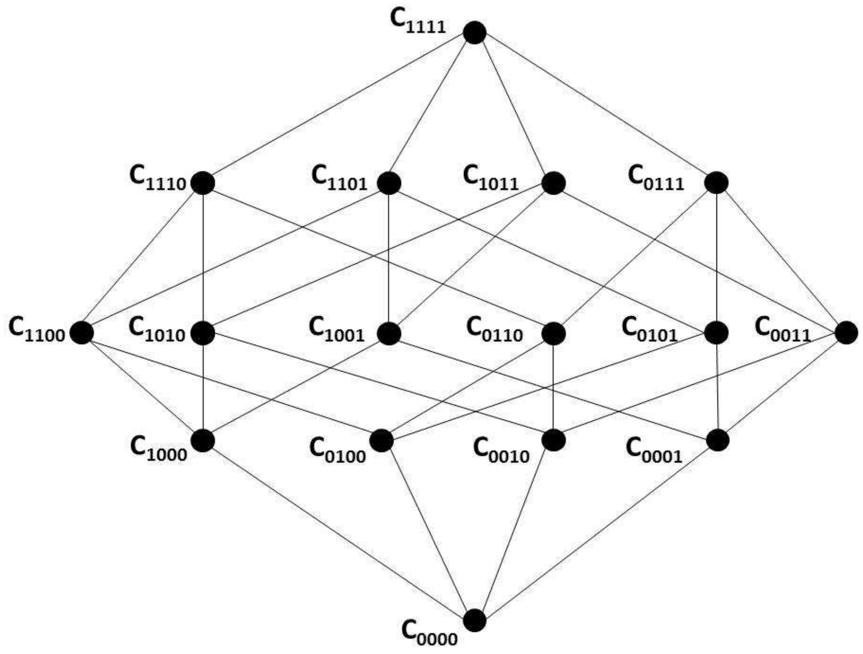


*Figure 6.1: Hasse Diagram for Four Attributes.*

Performing the ordering of the payoffs, Table 6.1 shows:

*Table 6.1: Possible Deterministic Consequences.*

| | $\vec{p}^{\,1}$ | $\vec{p}^{\,2}$ | $\vec{p}^{\,3}$ | $\vec{p}^{\,4}$ | $\vec{p}^{\,5}$ | $\vec{p}^{\,6}$ | $\vec{p}^{\,7}$ | $\vec{p}^{\,8}$ | $\vec{p}^{\,9}$ | $\vec{p}^{\,10}$ | $\vec{p}^{\,11}$ | $\vec{p}^{\,12}$ | $\vec{p}^{\,13}$ | $\vec{p}^{\,14}$ | $\vec{p}^{\,15}$ | $\vec{p}^{\,16}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p_1$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| $p_2$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| $p_3$ | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| $p_4$ | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |

The vectors were placed in order of preference, since the order of priorities were established by the decision-maker, therefore:

$$\vec{p}^{\,1} \prec \vec{p}^{\,2} \prec \vec{p}^{\,3} \prec \cdots \prec \vec{p}^{\,16} \qquad (6.2.2)$$

ii) *States of Nature*

The representative set of states of nature reflect three levels of scenarios:

$\Theta = \{\theta_1, \theta_2, \theta_3\}$, where:

$\theta_1$ — represents a favorable economic situation to finance the project implementation of the Museum of Sacred Art.

$\theta_2$ — represents an economic situation for the regular funding of the project.

$\theta_3$ — represents a bleak economy in which to finance the project.

iii) *Actions Set*

$\mathcal{A} = \{a_1, a_2, a_3\}$

- $a_1$ — Management and project execution moved forward with considerable boldness, with rigorous regarding scope, cost, time and quality.
- $a_2$ — Management and project execution happened in a neutral way.
- $a_3$ — Management and project execution were carried out in a relaxed manner, without strict control over scope, cost, time and quality.

iv) *The Decision Maker Preferences — The Utility Function*

Using the System for Educing Preferences - SEP, developed by Albuquerque (2011), the utility function of the manager was educed in the problem in question. The points obtained are shown in Figure 6.2 and Table 6.2.

The order of preference vectors can be observed in sequence 6.2.2, showing the worst consequence as $u(\vec{p}^{\,1}) = 0$ and the best, $u(\vec{p}^{\,16}) = 1$.

Intermediate values are determined by asking the decision-maker to go through a lottery with the aim of determining $\lambda_j$, ie:

$$\vec{p}^{\,j} \text{with probability } 1 \sim \begin{cases} \vec{p}^{\,16} & \text{with probability} \quad \lambda_j \\ \vec{p}^{\,1} & \text{with probability} \quad 1 - \lambda_j \end{cases}$$

Through the answer given by the decision-maker to the SEP system, which draws up scenarios like the lottery as above, where the range of the decision maker's utility function is in the interval $[0, 1]$, so for each scenario the value of $\lambda_j$ is given to reach a proposed situation on a scale of $\vec{p}^{\,1}$ to $\vec{p}^{\,16}$, which thereby determined the proceedings for the decision maker's preferences.

The points on the curve of the ABM decision-maker's utility function is shown in the Table 6.2:

*Table 6.2: Decision Maker's Utility Function.*

| Utility | $\vec{p}^{\,1}$ | $\vec{p}^{\,2}$ | $\vec{p}^{\,3}$ | $\vec{p}^{\,4}$ | $\vec{p}^{\,5}$ | $\vec{p}^{\,6}$ | $\vec{p}^{\,7}$ | $\vec{p}^{\,8}$ |
|---|---|---|---|---|---|---|---|---|
| $u(p)$ | 0,0000 | 0,0212 | 0,170 | 0,0212 | 0,0851 | 0,1221 | 0,0801 | 0,0851 |

| Utility | $\vec{p}^{\,9}$ | $\vec{p}^{\,10}$ | $\vec{p}^{\,11}$ | $\vec{p}^{\,12}$ | $\vec{p}^{\,13}$ | $\vec{p}^{\,14}$ | $\vec{p}^{\,15}$ | $\vec{p}^{\,16}$ |
|---|---|---|---|---|---|---|---|---|
| $u(p)$ | 0,1893 | 0,1893 | 0,3109 | 0,1893 | 0,4325 | 0,3920 | 0,7568 | 1,000 |

The curve plotted in Figure 6.2 refers to data from Table 6.2, that matches the profile of the project manager.

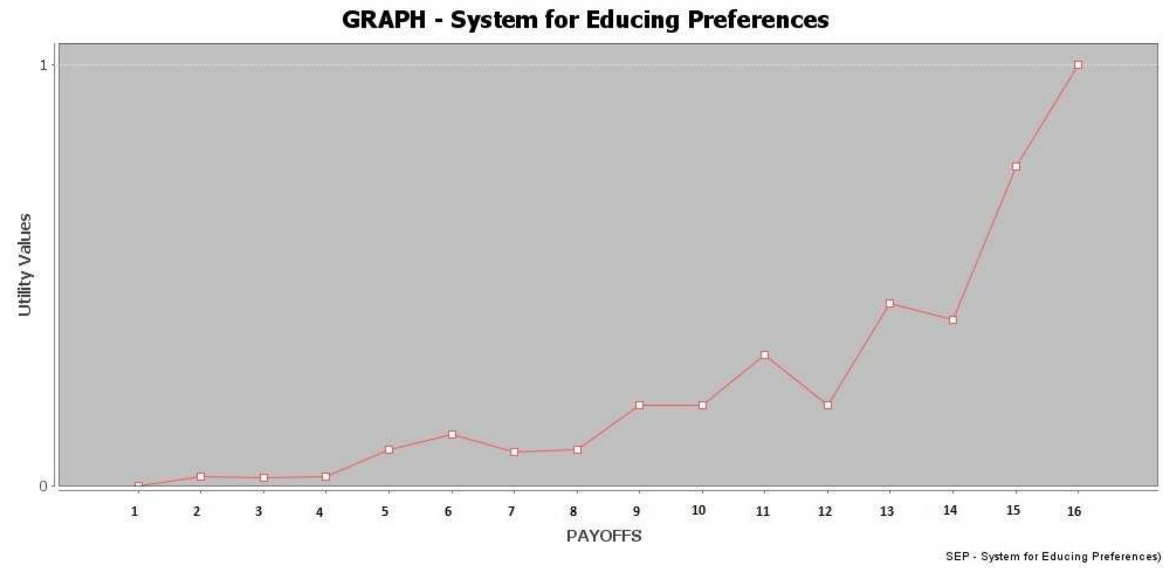


*Figure 6.2: Dispersion of the Manager's Utility Function ABM.*

Monotonicity utility curve discrepancies are shown in the Figure 6.2 are supposedly due to errors of introspection, according to Campello de Souza (2007b).

v) *Consequence Functions*

The consequence function result is the probability of getting a payoff $p$, since the nature chooses a state $\theta$ and the decision-maker choses the action $a$. The function can therefore be estimated from the experience of experts in the process of eduction or by means of historical data. For purposes of calculation of the probabilities of the consequence function result binomial distribution was used to simulate the data, as shown in Table 6.3:

*Table 6.3: Consequence Function $P(p|\theta, a)$.*

| $(\theta, a)$ | $\vec{p}^{\,1}$ | $\vec{p}^{\,2}$ | $\vec{p}^{\,3}$ | $\vec{p}^{\,4}$ | $\vec{p}^{\,5}$ | $\vec{p}^{\,6}$ | $\vec{p}^{\,7}$ | $\vec{p}^{\,8}$ | $\vec{p}^{\,9}$ | $\vec{p}^{\,10}$ | $\vec{p}^{\,11}$ | $\vec{p}^{\,12}$ | $\vec{p}^{\,13}$ | $\vec{p}^{\,14}$ | $\vec{p}^{\,15}$ | $\vec{p}^{\,16}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(\theta_1, a_1)$ | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,11 | 0,01 | 0,02 | 0,18 | 0,21 | 0,06 | 0,13 | 0,19 | 0,06 | 0,02 | 0,00 |
| $(\theta_1, a_2)$ | 0,00 | 0,00 | 0,00 | 0,02 | 0,01 | 0,20 | 0,05 | 0,10 | 0,19 | 0,14 | 0,16 | 0,03 | 0,08 | 0,01 | 0,00 | 0,00 |
| $(\theta_1, a_3)$ | 0,00 | 0,00 | 0,02 | 0,13 | 0,06 | 0,12 | 0,19 | 0,21 | 0,06 | 0,02 | 0,18 | 0,00 | 0,01 | 0,00 | 0,00 | 0,00 |
| $(\theta_2, a_1)$ | 0,00 | 0,00 | 0,00 | 0,03 | 0,01 | 0,21 | 0,06 | 0,12 | 0,18 | 0,12 | 0,18 | 0,02 | 0,06 | 0,01 | 0,00 | 0,00 |
| $(\theta_2, a_2)$ | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,05 | 0,00 | 0,01 | 0,11 | 0,18 | 0,02 | 0,21 | 0,23 | 0,13 | 0,05 | 0,01 |
| $(\theta_2, a_3)$ | 0,00 | 0,02 | 0,06 | 0,19 | 0,13 | 0,06 | 0,21 | 0,18 | 0,02 | 0,01 | 0,11 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| $(\theta_3, a_1)$ | 0,02 | 0,08 | 0,18 | 0,22 | 0,24 | 0,01 | 0,15 | 0,07 | 0,00 | 0,00 | 0,03 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 |
| $(\theta_3, a_2)$ | 0,00 | 0,00 | 0,02 | 0,13 | 0,06 | 0,12 | 0,19 | 0,21 | 0,06 | 0,02 | 0,18 | 0,00 | 0,01 | 0,00 | 0,00 | 0,00 |
| $(\theta_3, a_3)$ | 0,00 | 0,00 | 0,00 | 0,01 | 0,00 | 0,20 | 0,04 | 0,09 | 0,20 | 0,16 | 0,15 | 0,04 | 0,09 | 0,01 | 0,00 | 0,00 |

Utility of Consequence Function:

$$u(P(p|\theta, a)) = \sum_{p} v(p)P(p|\theta, a) \tag{6.2.3}$$

The values $v(p)$ are obtained from the utility function of payoffs in Table 6.2, while the values of $P(p|\theta, a)$ are taken from Table 6.3, resulting in Table 6.4.

*Table 6.4: Utility of Consequence Function $u(P(p|\theta, a))$.*

| State of Nature, Space Actions | $u(P(p\|\theta, a))$ |
|---|---|
| $(\theta_1, a_1)$ | 0,255 |
| $(\theta_1, a_2)$ | 0,197 |
| $(\theta_1, a_3)$ | 0,131 |
| $(\theta_2, a_1)$ | 0,188 |
| $(\theta_2, a_2)$ | 0,308 |
| $(\theta_2, a_3)$ | 0,097 |
| $(\theta_3, a_1)$ | 0,059 |
| $(\theta_3, a_2)$ | 0,131 |
| $(\theta_3, a_3)$ | 0,204 |

Calculation of the Loss Function $L(\theta, a)$, shown in Table 6.5, which is the negative consequence of the utility function in Table 6.4:

vi) *Set of Observations*

$\mathcal{X} = \{x_1, x_2, x_3\}$

As it was set the scale of observations and as $x_i$ can be seen, has

- $x_1$ — Periodic assessment of project status within the standards of excellence: low costs, shorter execution time and higher quality. This is an indication that the state of nature will have an optimistic scenario $\theta_1$.

*Table 6.5: Loss Function $L(\theta, a)$.*

| State of Nature, Space Actions | $L(\theta, a)$ |
|---|---|
| $(\theta_1, a_1)$ | $-0,255$ |
| $(\theta_1, a_2)$ | $-0,197$ |
| $(\theta_1, a_3)$ | $-0,131$ |
| $(\theta_2, a_1)$ | $-0,188$ |
| $(\theta_2, a_2)$ | $-0,308$ |
| $(\theta_2, a_3)$ | $-0,097$ |
| $(\theta_3, a_1)$ | $-0,059$ |
| $(\theta_3, a_2)$ | $-0,131$ |
| $(\theta_3, a_3)$ | $-0,204$ |

- $x_2$ — Evaluation of the project within acceptable standards. This is an indication that the state of nature will have a moderate scenario $\theta_2$.
- $x_3$ — Periodic assessment of project status outside the box, overspending, poor quality and project delay. This is an indication that the state of nature will have a worst case scenario $\theta_3$.

vii) *Likelihood Function*

$P(x|\theta)$ is the function that maps the set of observations seen by the decision-maker with all the states of nature, conditionally. Table 6.6 shows the estimate of the values of the likelihood function:

*Table 6.6: Likelihood Function $P(x|\theta)$.*

| observations | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|---|---|---|---|
| $x_1$ | 0,65 | 0,20 | 0,15 |
| $x_2$ | 0,15 | 0,70 | 0,15 |
| $x_3$ | 0,10 | 0,25 | 0,65 |

viii) *Function Risk*

From the calculation of the loss function and values of the likelihood function, one can calculate the risk:

$$R_d = \sum_{x} L(\theta, d(x))P(x|\theta) \tag{6.2.4}$$

Considering all possible combinations of decision making rules we have in Table 6.7:

*Table 6.7: Decision Making Rules.*

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_1$ | $a_1$ | $a_2$ | $a_3$ |
| $d_2$ | $a_1$ | $a_3$ | $a_2$ |
| $d_3$ | $a_2$ | $a_1$ | $a_3$ |
| $d_4$ | $a_2$ | $a_3$ | $a_1$ |
| $d_5$ | $a_3$ | $a_1$ | $a_2$ |
| $d_6$ | $a_3$ | $a_2$ | $a_1$ |
| $d_7$ | $a_1$ | $a_1$ | $a_2$ |
| $d_8$ | $a_1$ | $a_1$ | $a_3$ |
| $d_9$ | $a_2$ | $a_2$ | $a_1$ |

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_{10}$ | $a_2$ | $a_2$ | $a_3$ |
| $d_{11}$ | $a_3$ | $a_3$ | $a_1$ |
| $d_{12}$ | $a_3$ | $a_3$ | $a_2$ |
| $d_{13}$ | $a_2$ | $a_1$ | $a_1$ |
| $d_{14}$ | $a_3$ | $a_1$ | $a_1$ |
| $d_{15}$ | $a_1$ | $a_2$ | $a_2$ |
| $d_{16}$ | $a_3$ | $a_2$ | $a_2$ |
| $d_{17}$ | $a_1$ | $a_3$ | $a_3$ |
| $d_{18}$ | $a_2$ | $a_3$ | $a_3$ |

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_{19}$ | $a_1$ | $a_2$ | $a_1$ |
| $d_{20}$ | $a_1$ | $a_3$ | $a_2$ |
| $d_{21}$ | $a_2$ | $a_3$ | $a_2$ |
| $d_{22}$ | $a_2$ | $a_1$ | $a_2$ |
| $d_{23}$ | $a_3$ | $a_1$ | $a_3$ |
| $d_{24}$ | $a_3$ | $a_2$ | $a_3$ |
| $d_{25}$ | $a_1$ | $a_1$ | $a_1$ |
| $d_{26}$ | $a_2$ | $a_2$ | $a_2$ |
| $d_{27}$ | $a_3$ | $a_3$ | $a_3$ |

Thus, considering the set of decision making rules, we can calculate the risk function in Table 6.8, for each value of $\theta$, considering the loss function $L(\theta, d)$ and the likelihood function $P(x|\theta)$, in Tables 6.5 and 6.6, respectively.

*Table 6.8: Risk Function $R_d$.*

| decisions | $\theta_1$ | $\theta_2$ | $\theta_3$ | decisions | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|---|---|---|---|---|---|---|---|
| $d_1$ | $-0,225$ | $-0,258$ | $-0,171$ | $d_{15}$ | $-0,235$ | $-0,290$ | $-0,124$ |
| $d_2$ | $-0,221$ | $-0,142$ | $-0,142$ | $d_{16}$ | $-0,154$ | $-0,276$ | $-0,139$ |
| $d_3$ | $-0,199$ | $-0,192$ | $-0,160$ | $d_{17}$ | $-0,212$ | $-0,110$ | $-0,189$ |
| $d_4$ | $-0,193$ | $-0,142$ | $-0,102$ | $d_{18}$ | $-0,176$ | $-0,128$ | $-0,196$ |
| $d_5$ | $-0,166$ | $-0,192$ | $-0,120$ | $d_{19}$ | $-0,143$ | $-0,272$ | $-0,077$ |
| $d_6$ | $-0,163$ | $-0,258$ | $-0,091$ | $d_{20}$ | $-0,221$ | $-0,145$ | $-0,142$ |
| $d_7$ | $-0,246$ | $-0,206$ | $-0,106$ | $d_{21}$ | $-0,184$ | $-0,154$ | $-0,149$ |
| $d_8$ | $-0,236$ | $-0,174$ | $-0,153$ | $d_{22}$ | $-0,209$ | $-0,224$ | $-0,113$ |
| $d_9$ | $-0,206$ | $-0,290$ | $-0,084$ | $d_{23}$ | $-0,156$ | $-0,161$ | $-0,167$ |
| $d_{10}$ | $-0,187$ | $-0,276$ | $-0,178$ | $d_{24}$ | $-0,144$ | $-0,245$ | $-0,186$ |
| $d_{11}$ | $-0,150$ | $-0,110$ | $-0,109$ | $d_{25}$ | $-0,255$ | $-0,188$ | $-0,059$ |
| $d_{12}$ | $-0,141$ | $-0,128$ | $-0,157$ | $d_{26}$ | $-0,197$ | $-0,308$ | $-0,131$ |
| $d_{13}$ | $-0,217$ | $-0,206$ | $-0,066$ | $d_{27}$ | $-0,131$ | $-0,097$ | $-0,204$ |
| $d_{14}$ | $-0,175$ | $-0,174$ | $-0,073$ | | | | |

ix) *Prior Knowledge*

The a prior probability $\pi(\theta)$ was estimated by the Decision-Maker ABM, due to experience in similar projects that in 85% of the time the general conditions for financing are good or reasonable. The prior probabilities $\pi(\theta)$ was educed according to Table 6.9:

*Table 6.9: Prior Probabilities $\pi(\theta)$.*

| $\theta$ | $\pi(\theta)$ |
|---|---|
| $\theta_1$ | 0,60 |
| $\theta_2$ | 0,25 |
| $\theta_3$ | 0,15 |

x) *Bayes Decision Rule*

With the information of the a prior probability distribution, we calculate the Bayes risk, as:

$$r_d = \sum_{\theta} \pi(\theta) R_d(\theta) \tag{6.2.5}$$

The calculation of the Bayes risk is shown in the Table 6.10:

*Table 6.10: Bayes Risk $r_d$.*

| decisions | $x_1$ | $x_2$ | $x_3$ | $\theta_1$ | $\theta_2$ | $\theta_3$ | $r_d$ |
|---|---|---|---|---|---|---|---|
| $d_1$ | $a_1$ | $a_2$ | $a_3$ | $-0,225$ | $-0,258$ | $-0,171$ | $-0,225$ |
| $d_2$ | $a_1$ | $a_3$ | $a_2$ | $-0,221$ | $-0,142$ | $-0,142$ | $-0,190$ |
| $d_3$ | $a_2$ | $a_1$ | $a_3$ | $-0,199$ | $-0,192$ | $-0,160$ | $-0,191$ |
| $d_4$ | $a_2$ | $a_3$ | $a_1$ | $-0,193$ | $-0,142$ | $-0,102$ | $-0,166$ |
| $d_5$ | $a_3$ | $a_1$ | $a_2$ | $-0,166$ | $-0,192$ | $-0,120$ | $-0,166$ |
| $d_6$ | $a_3$ | $a_2$ | $a_1$ | $-0,163$ | $-0,258$ | $-0,091$ | $-0,176$ |
| $d_7$ | $a_1$ | $a_1$ | $a_2$ | $-0,246$ | $-0,206$ | $-0,106$ | $-0,215$ |
| $d_8$ | $a_1$ | $a_1$ | $a_3$ | $-0,236$ | $-0,174$ | $-0,153$ | $-0,208$ |
| $d_9$ | $a_2$ | $a_2$ | $a_1$ | $-0,206$ | $-0,290$ | $-0,084$ | $-0,209$ |
| $d_{10}$ | $a_2$ | $a_2$ | $a_3$ | $-0,187$ | $-0,276$ | $-0,178$ | $-0,208$ |
| $d_{11}$ | $a_3$ | $a_3$ | $a_1$ | $-0,150$ | $-0,110$ | $-0,109$ | $-0,134$ |
| $d_{12}$ | $a_3$ | $a_3$ | $a_2$ | $-0,141$ | $-0,128$ | $-0,157$ | $-0,140$ |
| $d_{13}$ | $a_2$ | $a_1$ | $a_1$ | $-0,217$ | $-0,206$ | $-0,066$ | $-0,192$ |
| $d_{14}$ | $a_3$ | $a_1$ | $a_1$ | $-0,175$ | $-0,174$ | $-0,073$ | $-0,159$ |
| $d_{15}^*$ | $a_1$ | $a_2$ | $a_2$ | $-0,235$ | $-0,290$ | $-0,124$ | $-0,232$ |
| $d_{16}$ | $a_3$ | $a_2$ | $a_2$ | $-0,154$ | $-0,276$ | $-0,139$ | $-0,183$ |
| $d_{17}$ | $a_1$ | $a_3$ | $a_3$ | $-0,212$ | $-0,110$ | $-0,189$ | $-0,183$ |
| $d_{18}$ | $a_2$ | $a_3$ | $a_3$ | $-0,176$ | $-0,128$ | $-0,196$ | $-0,167$ |
| $d_{19}$ | $a_1$ | $a_2$ | $a_1$ | $-0,243$ | $-0,272$ | $-0,077$ | $-0,226$ |
| $d_{20}$ | $a_1$ | $a_3$ | $a_2$ | $-0,221$ | $-0,145$ | $-0,142$ | $-0,190$ |
| $d_{21}$ | $a_2$ | $a_3$ | $a_2$ | $-0,184$ | $-0,154$ | $-0,149$ | $-0,171$ |
| $d_{22}$ | $a_2$ | $a_1$ | $a_2$ | $-0,209$ | $-0,224$ | $-0,113$ | $-0,198$ |
| $d_{23}$ | $a_3$ | $a_1$ | $a_3$ | $-0,156$ | $-0,161$ | $-0,167$ | $-0,159$ |
| $d_{24}$ | $a_3$ | $a_2$ | $a_3$ | $-0,144$ | $-0,245$ | $-0,186$ | $-0,176$ |
| $d_{25}$ | $a_1$ | $a_1$ | $a_1$ | $-0,255$ | $-0,188$ | $-0,059$ | $-0,209$ |
| $d_{26}$ | $a_2$ | $a_2$ | $a_2$ | $-0,197$ | $-0,308$ | $-0,131$ | $-0,215$ |
| $d_{27}$ | $a_3$ | $a_3$ | $a_3$ | $-0,131$ | $-0,097$ | $-0,204$ | $-0,133$ |

Thus, the decision rule that minimizes the Bayes risk is the one that has the maximum utility. That rule is $d_{15}^* = -0,232$. It may be noted that this Table 6.10 represents the lowest risk among all risks calculated, corresponding to the following situation shown in Table 6.11:

*Table 6.11: Choose the Best Decision Making Rule.*

| decisions | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| $d_{15}^*$ | $a_1$ | $a_2$ | $a_2$ |

Thus, this is the best action corresponding to “Management and project execution can move forward with considerable boldness, with rigorous scope, cost, time and quality”. The action $a_1$ should be chosen if the project manager observes $x_1$. On the other hand, in the case where the observation is $x_2$ or $x_3$ the best action would be $a_2$ “Management and project execution in a neutral way”. However, these choices are characterized by the profile by which the decision-maker was chosen. From the data educed the decision-maker through the SEP system is plotted as shown by the corresponding curve in Figure 6.2. It is observed by the curvature of the graph that the decision-maker has the characteristics of risk propensity. The layout of the curve is roughly like the one shown in Figure 2.3 in subsection 2.2.6.

By type of project where the risks are high, the project manager can choose a type of contract as the Public-Private Partnerships, to share risks more efficiently according to Palma (2009). One can also use the principles of the theory of the main agent.

In sequential decision-making the next decision would take into account the new scenario recalculated for all risks associated with the new situation.

# 7 CONCLUSIONS, COMMENTS AND SUGGESTIONS

## 7.1 Conclusions

In the case studies presented in both the Management and Operation of a Nuclear Power Plant project and the implantation of a Museum of Sacred Art, we calculated the risks associated with the decisions, but also the Bayes risk, which represents the lowest risk and that maximizes the utility in the decision, making it the best choice.

Risk management for these complex projects was analyzed from the perspective of Game Theory and Decision Theory. During the implementation phase and at the time of the main design decisions, a system was proposed, according to a strategic cooperative game, with the principal and the agent taking an advance notice before each decision, adopting a game (project implementation) of asymmetric information due to uncertainty, and the complexity of the relevant project. It was considered that each player can still reconsider his plan of action at every moment of the game at the moment of decision, resulting in project management being conducted as an extensive game.

The systematic proposal sought to detail so that the mathematics of the theories presented could be implemented in practice through a set of project information derived from a database in the deployment phase of the project.

The relevant sections of the proposed systematization presented in this dissertation are listed below:

- The utility function is the essence of Decision Theory and reflects the preference of the decision-maker in times of uncertainty. This preference about the tradeoffs and uncertainties are taken into account in the calculations of associated risks.

- For each calculated risk, the probability of occurrence of events and their impacts is already included. Likewise, are included uncertainties and tradeoffs.

- A study of improved decision-maker's utility function may facilitate the analysis of tradeoffs for complex projects, for example Runtime $\times$ Cost, Quality $\times$ Cost , to support decision-making for both the principal as for the agent, thus reducing the effects of asymmetry of information, creating a greater chance of success in project implementation.

- The reduction of the effects of information asymmetry is due to the fact that we calculate the Bayes risk associated with the set of actions arranged for the decision maker as well as a sample and reference profiles for aversion, indifference and propensity towards risk.

- The proposal promotes the systematic identification, classification and quantification of risks inherent in the implementation of complex projects, facilitating the sharing of risks, as well as the construction of risk matrices, facilitating the construction of contractual terms established between the principal and the agent.

- The calculation of the minimum cumulative metric of the Bayes risk allows it to be used as a reference line for optimal project risks. This providing an optimal policy for managing the project.

- The systematic decision-making system presented in this work considers the calculation of all the risks associated with the decisions for each scenario. Thus, the project manager has an order of magnitude from lower risk to higher risk for each scenario.

- A metrics is generated to measure, at any stage of the project, the level of cumulative risk, through the parameter of Minimum Cumulative Risk of Bayes.

- Even if the decision-maker makes a wrong decision, with this methodology the distance error for the optimal choices can be calculated and compared, calculating the Bayes risk of wrong decision and Bayes risk for the optimal decision.

- Using the strategy of the cumulative sum of the minimum Bayes risk, the project manager has to guarantee an optimal policy decisions in managing complex projects.

The Minimum Bayes Risk Accumulated provides the project manager a degree of accuracy for cumulative decisions, reflecting a level of risk in relation to decisions to step N of the project. Of all possible N-tuples generated, the measure of risk is the

Bayes minimum accumulated that provides the least weight, because it is based on those decisions, every step of the project, which carry the lowest risk. The sequence of lower risk at each stage is associated with a better strategy and therefore the optimal policy.

Considering the results obtained and the concepts exposed, window is opened for serious future work regarding the study of imprecise probabilities to reduce uncertainty in decision making. Another vision is to further explore the concepts of the economic applications of Game Theory to solve optimal decision policies.

## 7.2 Comments and Suggestions

This work could be accomplished by using only the mathematical constructs of the Theory of Decision. The inclusion of the mathematical framework of Game Theory with data, however, is to enrich the moment of decision.

In Game Theory the concept of principal-agent is that both players want to maximize their utility. Therefore, the intentions of the agent shown in their actions will allow such information to be viewed by the decision maker.

Suggestions for future work are:

- In sequential decisions consideration of the decision maker's utility function when time-varying during the implementation of complex projects.

- Working in the reverse process. With more project information, a simulation could be made to choose the best profile of the project manager. That would have a utility function consistent with the requirements of complex project, to produce lower accumulated risks.

- Analyze risks in decision making of principal and agent simultaneously.

- Include elements of game theory in payoffs such as: Incentive Compatibility Rate (ICR) related to the Principal to propose certain additional benefits as a way to encourage producing the desired results and the moral hazard. This refers to the fact that the efforts made by the agent dedicated to the task can not be freely observable by the principal and thus cause the monitoring problems.

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